算例 1-009

框架 –框架构件施加 预应力

算例描述

通过一个有抛物线预应力钢筋和在两端有不同偏心的简支混凝土梁，测试了 SAP2000 的梁预应力计算。该梁承受自重荷载和预应力荷载。将得到的弯矩和梁中挠度与独立的手算结果进行了对比。

SAP2000 有两种方式模拟预应力效应。一种方式模拟施加在结构上的作为外荷载的预应力。另一种方式模拟预应力筋。本例对这两种方式都进行验证。

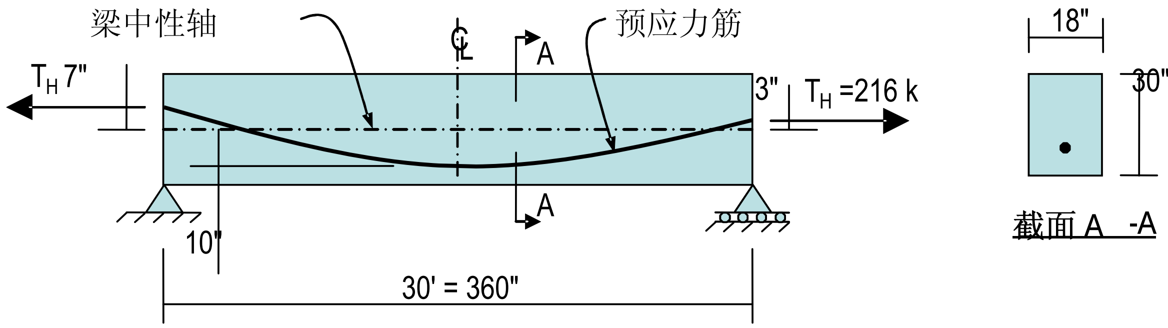
在 SAP2000 中，该梁通过两端框架单元模拟，以使梁中点处存在节点，这样可以求得梁中节点处的位移。

分析中使用三个不同的模型。模型 A 具有模拟为荷载的预应力。模型 B 和 C 的预应力体现在单元上。模型 B 预应力筋最大剖分尺寸为 60 英寸。模型 C 的预应力筋最大剖分尺寸为 12 英寸。

本例中包括了错动和曲线预应力损失效应，以及混凝土梁的弹性缩短。预应力筋只从左侧施加预应力。

重要提示：本例考虑了剪切变形。

几何特性和荷载



材料属性

E = 3600 k/in²

v = 0.2

G= 1500 k/in²

3

截面属性

b = 18 in

d = 30 in

A = 540 in²

I = 40,500 in⁴

A_v = 450 in² (剪切面积)

荷载

A=1.5in²   E=29000k/in²   V=0.3

预应力筋注释：

1. T 是预应力筋在预应力损失之前的拉力分量。
2. 筋仅从左侧张拉。
3. 图中显示了梁的左、右、中部筋距中性轴的距离。
4. 筋形状为抛物线。
5. 曲线摩擦损失系数为 0.15。
6. 错动摩擦损失系数为 0.0001 / 英寸。
7. 考虑摩擦和梁弹性缩短引起的损失。

所测试的**SAP2000** 技术要点：

- 有抛物线预应力钢筋和两端有不同偏心的简支混凝土预应力梁的建模
- 用荷载模拟预应力钢筋
- 用单元模拟预应力钢筋
- 预应力损失

结果比较

采用 Cook and Young 1985 一书第 244 页的基本原理和单位力法手算得出独立结果。

重要提示：手工计算时，假定作用于梁上的分布荷载为均布的。因此并不期望 SAP2000 和手算结果完全相同。

模型	输出参数	**SAP2000**	独立结果	差值百分比
A-通过荷载	U_z (中心) in	0.17007	0.16564	+2.7%
B- 单元 (60")		0.16282		-1.7%
C- 单元 (12")		0.16579		+0.1%
A- 通过荷载	M_y (中心) kip-in	-2059.6	-2004.3	+2.8%
B- 单元 (60")		-2007.8		+0.2%
C- 单元 (12")		-1997.9		-0.3%

计算模型文件: Example 1-009a, Example 1-009b, Example 1-009c

结论

SAP2000 的结果和独立计算的结果的对比在可接受范围。

手算过程

Calculate total center deflection of beam caused by prestress

$$\Delta_{total} = \Delta_{bending} + \Delta_{shear}$$

$$\Delta_{total} = \frac{TL^2}{96EI}(10d_c - d_i - d_j) + \frac{T}{2GA_v}(d_i + d_j + 2d_c)$$

$$= \frac{216 \times 360^2}{96 \times 4320 \times 40500}((10 \times 10) - 7 - 3)$$

$$+ \frac{216}{2 \times 1800 \times 450}(7 + 3 + (2 \times 10))$$

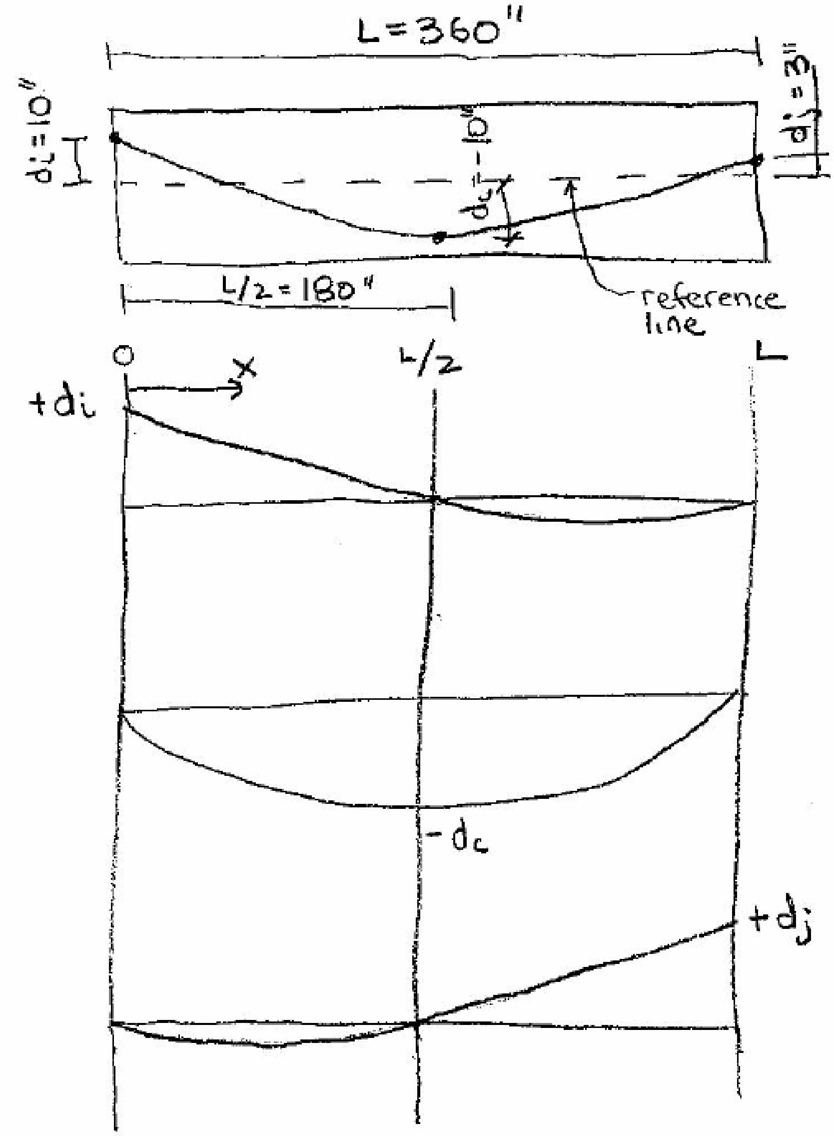
$$= 0.15 + 0.004$$

$$\therefore \Delta_{total} = \underline{\underline{0.154 \text{ in}}} \uparrow \text{ due to prestress alone}$$

手

HAND CALCULATION

1. Use parabolic shape functions to derive equations for tendon drape and slope.



$d(x)$ = cable drape measured positive upward from reference line

$$S(x) = \text{cable slope} = \frac{d}{dx} d(x)$$

$$d(x) = d_i \left(x - \frac{L}{2}\right)(x - L)\frac{2}{L^2} - d_c (x - 0)(x - L)\frac{4}{L^2} + d_j (x - 0)\left(x - \frac{L}{2}\right)\frac{2}{L^2}$$

$$d(x) = d_i \left(x^2 - \frac{3L}{2}x + \frac{L^2}{2}\right)\frac{2}{L^2} - d_c \left(x^2 - Lx\right)\frac{4}{L^2} + d_j \left(x^2 - \frac{Lx}{2}\right)\frac{2}{L^2}$$

$$d(x) = \left(\frac{2d_i - 4d_c + 2d_j}{L^2}\right)x^2 - \left(\frac{3d_i - 4d_c + d_j}{L}\right)x + d_i$$

$$S(x) = \frac{d}{dx} d(x) = \left(\frac{4d_i - 8d_c + 4d_j}{L^2}\right)x - \left(\frac{3d_i - 4d_c + d_j}{L}\right)$$

$$S(0) = -\left(\frac{3d_i - 4d_c + d_j}{L}\right)$$

$$S(L) = \frac{4d_i - 8d_c + 4d_j - 3d_i + 4d_c - d_j}{L} = \frac{d_i - 4d_c + 3d_j}{L}$$

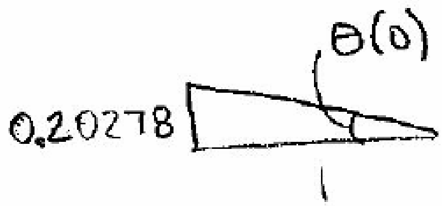
2. Calculate end slopes

$$S(0) = -\left(\frac{3\times10 - 4\times(-10) + 3}{360}\right) = -0.20278 \text{ in/in}$$

$$S(L) = \frac{10 - 4\times(-10) + 3\times3}{360} = 0.16389 \text{ in/in}$$

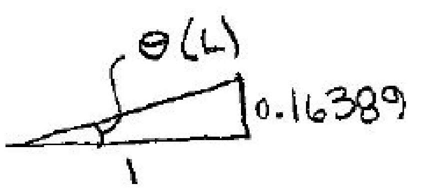
3. Calculate end angles

At L= 0":



$$\theta(0) = \text{Atan}(0.20278)$$
$$\theta(0) = 11.463°$$

At L= 360":



$$\theta(L) = \text{Atan}(0.16389)$$
$$\theta(L) = 9.307°$$

4. Determine friction loss coefficients at $L = 360''$

$$\text{Curvature Loss Coefficient} = \frac{0.15 \times (11.463^\circ + 9.307^\circ) \times \pi}{180}$$

$$= 0.05438$$

$$\text{Wobble Loss Coefficient} = 0.0001 \times 360'' = 0.036$$

5. Determine axial end forces before elastic shortening occurs

At $L = 0''$: $F_A = 220 \cos(11.463^\circ)$

$F_A = 215.612^k$

At $L = 360''$:

$F_A = 220(1 - 0.05438 - 0.036)\cos(9.307^\circ)$

$F_A = 197.482^k$

6. Estimate tension loss from axial shortening

P = average axial force

$$P = \frac{215.612 + 197.482}{2} = 206.5^k$$

$$\Delta_{conc} = \frac{PL}{E_{conc} A_{conc}}$$

$$\Delta_{conc} = \frac{206.5 \times 360}{3600 \times 18 \times 30} = 0.0382 \text{ in}$$

$$P_{tendon} = \frac{E_{tendon} A_{tendon} \Delta_{conc}}{L}$$

$$P_{tendon} = \frac{29000 \times 1.5 \times 0.0382}{360} = 4.62^k$$

Assume 4.6^k loss from axial shortening

$$\text{Stress loss} = \frac{4.6}{1.5} = 3.067 \text{ Ksi}$$

This loss is specified in Model A where the prestress is represented as loads.

7. Calculate end forces at L = 0"

$F_A = (220-4.6)\cos(11.463°) = 211.103^k \rightarrow$

$F_V = (220-4.6)\sin(11.463°) = 42.808^k \downarrow$

$M = 10F_A = 211.103 \times 10 = 2111.03$ k-in

8. Calculate end forces at L = 360"

$F_A = [\{220(1-0.05438-0.036)\}-4.6]\cos(9.307°)$

$F_A = 192.943^k \leftarrow$

$F_V = [\{220(1-0.05438-0.036)\}-4.6]\sin(9.307°)$

$F_V = 31.620^k \downarrow$

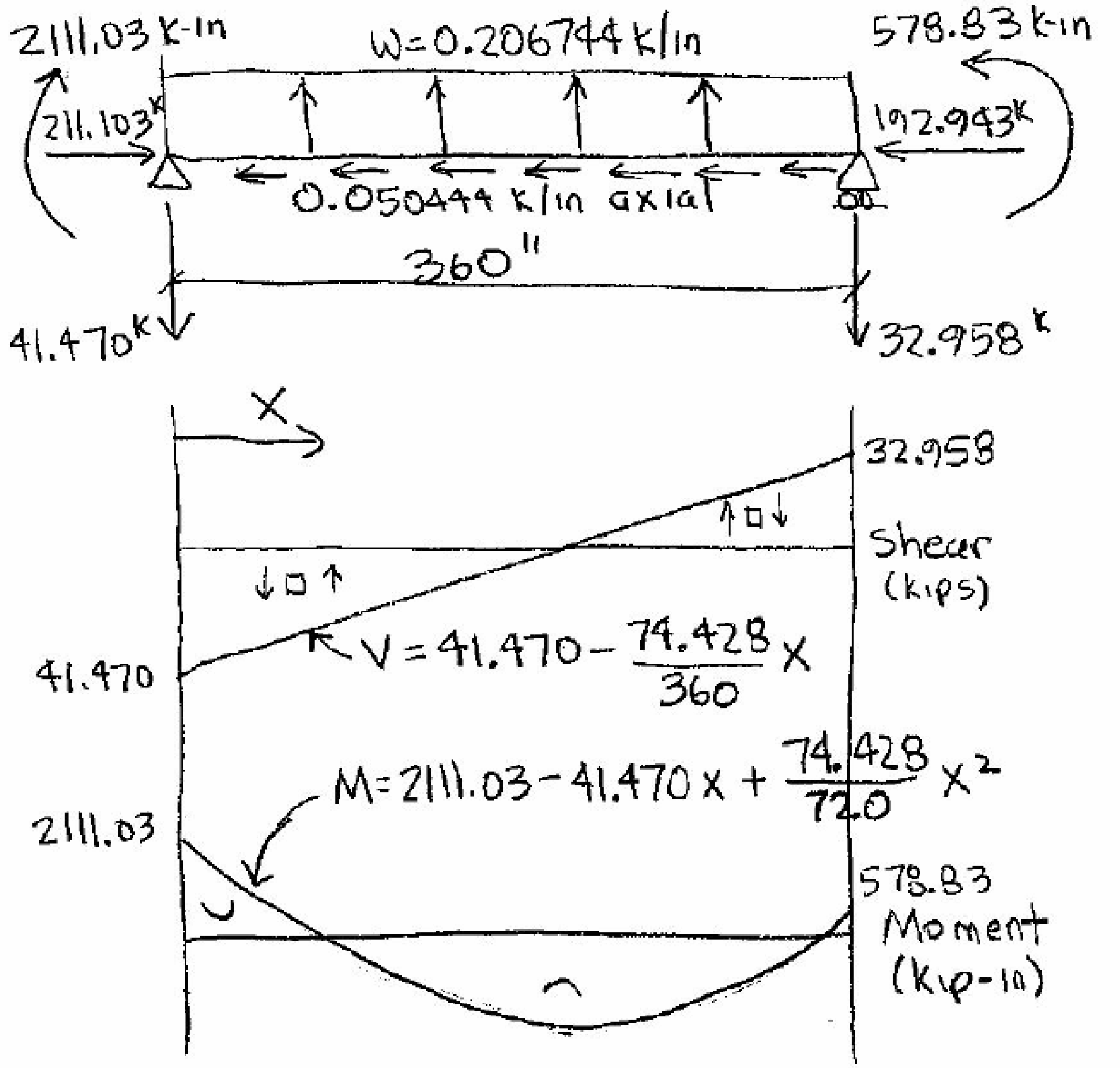
$M = 3F_A = 3 \times 192.943 = 578.83$ k-in

9. Determine magnitude of upward distributed load due to prestress. For simplicity in this hand calculation assume the distributed load is uniform.

$$\omega = \frac{(F_V @ L=0) + (F_V @ L=360)}{L} = \frac{42.808+31.620}{360}$$

$\omega = 0.206744$ K/in $\uparrow$

10. Plot equivalent loading on beam. Note that assumed uniform upward load from prestress yields calculated vertical reactions that are slightly different from previously calculated end forces



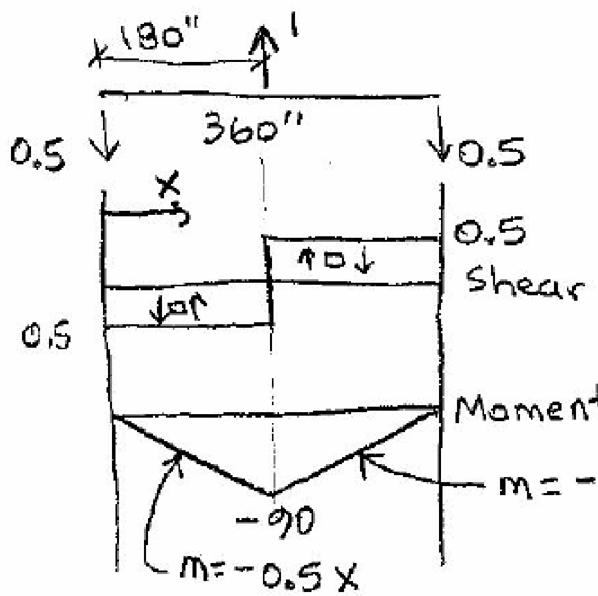
11. Calculate midspan deflection caused by prestress alone considering flexural and shear deformations.

Use $\Delta = \int \frac{mM}{EI} dx + \int \frac{vV}{GA_v} dx$

$$EI = \frac{3600 \times 18 \times 30^3}{12} = 145{,}800{,}000$$

$$GA_v = 1500 \times \frac{5}{6} \times 18 \times 30 = 675{,}000$$

Virtual System:



Flexural Deformation:

$$\Delta_f = \frac{1}{EI}\int_0^{180}\left(-1055.515x + 20.735x^2 - \frac{74.428}{1440}x^3\right)dx$$

$$+\frac{1}{EI}\int_{180}^{360}\left(-379985.4 + 7464.6x - \frac{74.428}{4}x^2\right)dx$$

$$-\frac{1}{EI}\int_{180}^{360}\left(-1055.515x + 20.735x^2 - \frac{74.428}{1440}x^3\right)dx$$

$$\Delta_f = \frac{1}{EI}\left(\frac{-1055.515x^2}{2} + \frac{20.735x^3}{3} - \frac{74.428x^4}{5760}\right)\Bigg|_0^{180}$$

$$+\frac{1}{EI}\left(-379985.4x + \frac{7464.6x}{2} - \frac{74.428x^3}{12}\right)\Bigg|_{180}^{360}$$

$$-\frac{1}{EI}\left(\frac{-1055.515x^2}{2} + \frac{20.735x^3}{3} - \frac{74.428x^4}{5760}\right)\Bigg|_{180}^{360}$$

$$\Delta_f = -0.117279 + 0.276467 - 0.093035$$
$$-0.938236 + 3.3176 - 1.984747$$
$$+0.469118 - 0.8294 + 0.248093$$
$$+0.469118 - 2.211733 + 1.48856$$
$$-0.117279 + 0.276467 - 0.093035$$

$\Delta_f = 0.160679$ in ↑, flexural deformation

Shear Deformation:

$$\Delta_v = \frac{1}{GA_v}\int_0^{180}\left(\frac{41.47}{2} - \frac{74.428}{720}x\right)dx$$
$$+\frac{1}{GA_v}\int_{180}^{360}\left(-\frac{41.47}{2} + \frac{74.428}{720}x\right)dx$$

$$\Delta_v = \frac{1}{GA_v}\left(\frac{41.47x}{2} - \frac{74.428x^2}{1440}\right)\Bigg|_0^{180}$$
$$+\frac{1}{GA_v}\left(-\frac{41.47x}{2} + \frac{74.428x^2}{1440}\right)\Bigg|_{180}^{360}$$

$$\Delta_v = 0.005529 - 0.002481 - 0.011059 + 0.009924 + 0.005529 - 0.002481$$

$\Delta_v = 0.004961$ in ↑, shear deformation

Total Deformation:

$$\Delta_{total} = \Delta_f + \Delta_v = 0.160679 + 0.004961$$

$$\underline{\Delta_{total} = 0.1564 \text{ in } \uparrow}$$

12. Calculate center moment caused by prestress alone

$$M_{center} = 2111.03 - 41.47 \times 180 + \frac{74.428}{720} \times 180^2$$

$$M_{center} = -2004.3 \text{ k-in}$$