算例 6-008

连接单元 – 塑性 WEN 连接

问题描述

本例是一个单自由度系统，用于测试塑性 Wen 连接的性能。为连接单元定义了双线性力-变形特性，还定义了用于反映从初始刚度到屈服刚度变化角度的指数。采用非线性静力分析该连接单元推至正的 10in.位移。然后基于该分析工况的最终状态进行第二个非线性分析，将连接单元推至-10in.位移，推了 20in.。将不同变形下的连接单元力与定义的连接单元力-变形关系特性进行了比较。

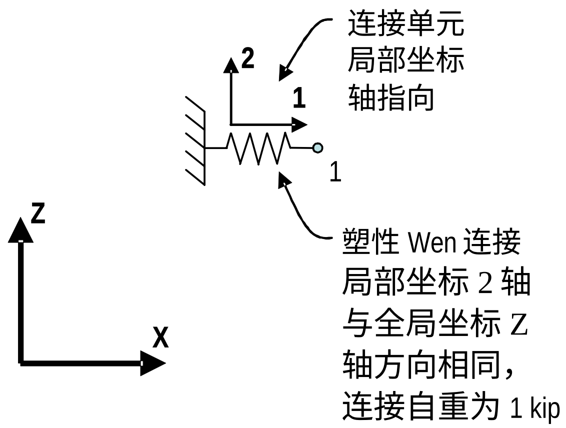
该 SAP2000 模型由一个单节点（标签为节点 1）和一个连接单元组成。在 XZ 平面内建立模型，分析中只有 U_Z 自由度是活动的。塑性 Wen 连接单元模拟为节点 1 的单节点连接单元。这意味着连接单元的一端接地，一端与节点 1 相连。将连接单元定位为其局部 2 轴与 Z 轴正向平行。这不是单节点连接单元的默认方向，所以采用了 SAP2000 中的高级局部坐标指定功能来获得所需的局部坐标方向。只定义了连接单元的 U_2 自由度。

因为只采用了非线性分析工况，所以只有连接单元的非线性特性与本例有关。连接单元的刚度为 100 k/in。其屈服力为 50 kips。初始刚度与屈服刚度之比为 0.1。换言之，屈服刚度为 10 k/in。控制初始刚度到屈服刚度转化角度的屈服指数为 1。

连接单元的重量为 1kip。这是作用于连接单元的唯一荷载，它是作为 Z 向的重力荷载施加的。

本例中采用了两个位移控制的非线性静力分析工况，名为 NLSTAT1 和 NLSTAT2。NLSTAT1 从零初始状态开始，将连接单元推至正 10in.的变形。NLSTAT2 从 NLSTAT1 结束时的状态开始，然后将连接单元从正 10in.变形推至负 10in.变形。

几何特性、属性和荷载



荷载

自重施加为
重力荷载

激活自由度

仅 U_z

连接属性 (U_2 DOF)

线性属性

没有被使用

剪力距离

不能应用于零长度连接单元

非线性属性

刚度, k	= 100 k/in
屈服内力, y	= 50 k
屈服比, r	= 0.1
屈服指数, e	= 1

连接单元的内力－变形曲线由下面等式进行定义：

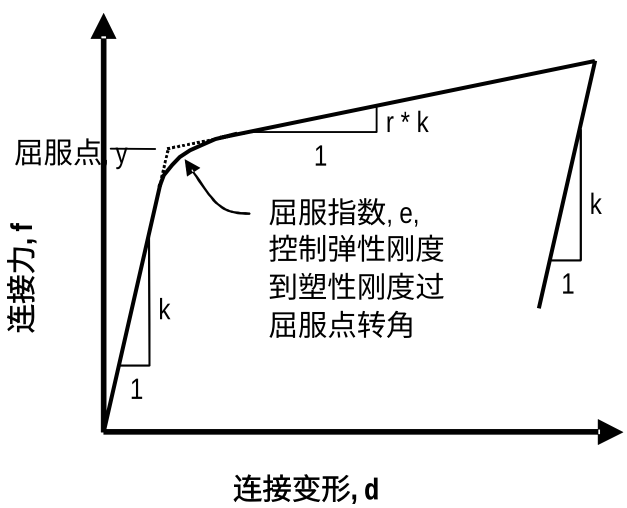
$f = r\,k\,d + (1 - r)\,y\,z$

此处

k, r 和 y 已经在上面定义过了
f = 内力
d = 变形
z = 奇异变量，此处 1 <= z<= 1, 初始 z 值为零，并且 z 值的变化遵循下面的公式：

$$\dot{z} = \frac{k}{y}\dot{d}\,(1-|z|^{e}) \quad 若\ \dot{d}z > 0$$

$$\dot{z} = \frac{k}{y}\dot{d} \quad 否则$$



所测试的 SAP2000 技术要点：

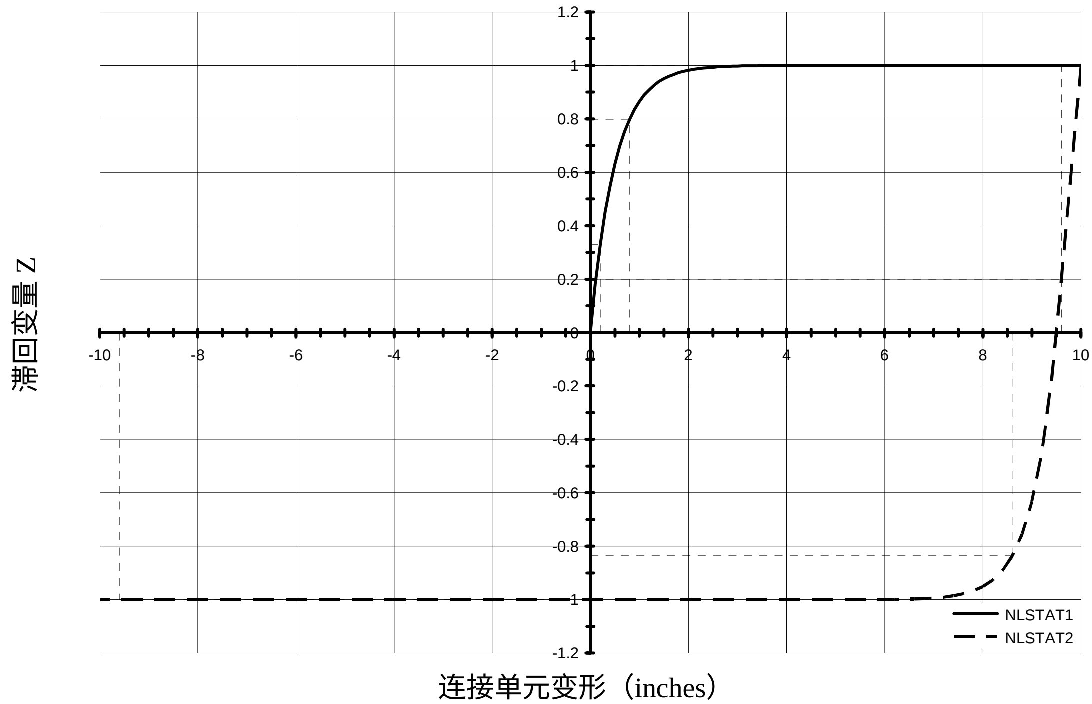
- 塑性 Wen 连接
- 位移控制的非线性静力分析
- 连接单元局部坐标指定
- 连接单元重力荷载

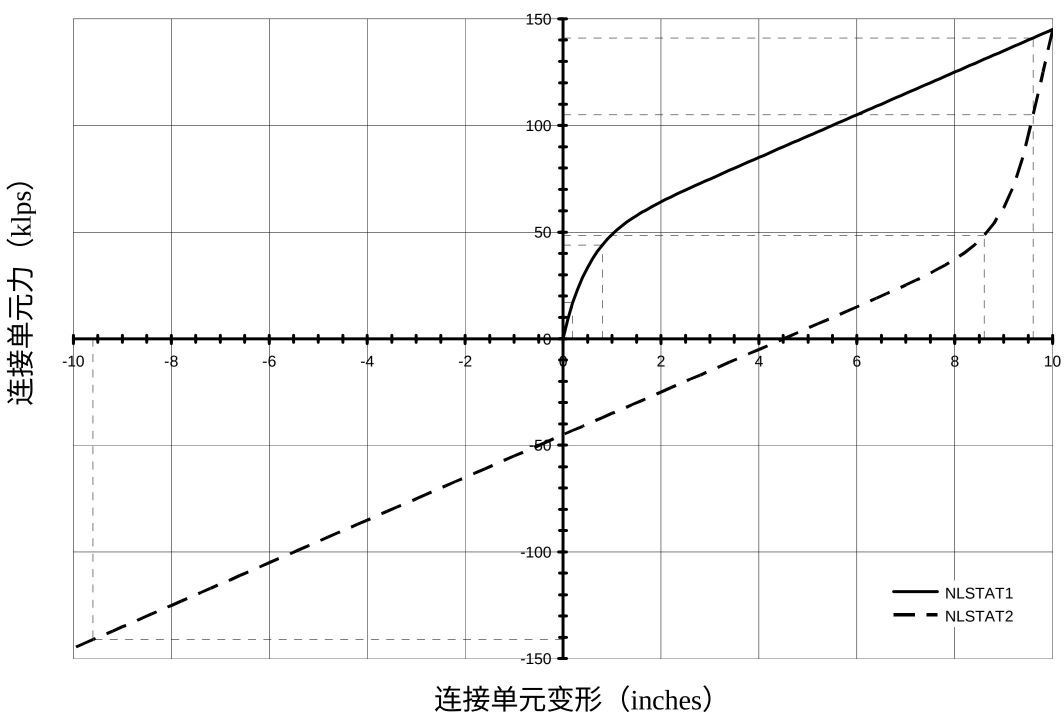
结果比较

如上页的图所示，可以通过定义的连接单元力-变形关系得到独立结果。

输出参数	连接单元变形	分析工况	SAP2000	手算结果	差值百分比
特定位移作用下指定分析工况的连接单元力 kips	0.20 in	NLSTAT1	16.836	16.836	0%
	0.80 in	NLSTAT1	43.915	43.915	0%
	9.60 in	NLSTAT1	141.000	141.000	0%
	9.60 in	NLSTAT2	105.000	105.000	0%
	8.60 in	NLSTAT2	48.438	48.438	0%
	-9.60 in	NLSTAT2	-141.000	-141.000	0%

下图绘出了连接单元变形和滞回变量 z 的关系，并给出了连接单元变形和连接单元力的关系。还绘出了分析工况 NLSTAT1 和 NLSTAT2 的结果。


滞回变量 Z
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-10
-8
-6
-4
-2
0
2
4
6
8
10
NLSTAT1
NLSTAT2
连接单元变形（inches）


连接单元力（klps）
150
100
50
0
-50
-100
-150
-10
-8
-6
-4
-2
0
2
4
6
8
10
NLSTAT1
NLSTAT2
连接单元变形（inches）

计算模型文件: Example 6-008

结论

SAP2000 结果与独立结果完全吻合。

手算过程

f = link force $= r k d + (1-r) Y z$

d = link deformation

k = link initial stiffness

Y = link yield force

r = link yield ratio = $\dfrac{\text{yielded stiffness}}{\text{initial stiffness}}$

z = internal hysteretic variable that is initially zero and evolves according to the following differential equation:

$$\dot{z} = \frac{k}{Y}\,\dot{d}\,(1-|z|^{e}) \quad \text{if } \dot{d}z > 0$$

$$\dot{z} = \frac{k}{Y}\,\dot{d} \quad \text{otherwise}$$

e = exponent used in definition of z and is equal to 1 for this example

$$\dot{d}z > 0 \text{ if } \begin{cases} z > 0 \text{ and } d \text{ is increasing} \\ z < 0 \text{ and } d \text{ is decreasing} \end{cases}$$

First find an expression for z when $\dot{d}z > 0$ and z is positive

$$\dot{z} = \frac{k}{y}\dot{d}\left(1 - |z|^{e}\right) = \frac{k}{y}\dot{d}\left(1 - |z|\right) \text{ since } e = 1$$

$$\dot{z} = \dot{d}\frac{k}{y} - \dot{d}\frac{k}{y}|z|$$

$$\dot{z} + \dot{d}\frac{k}{y}|z| = \dot{d}\frac{k}{y}$$

Find homogeneous solution for $\dot{z} + \dot{d}\frac{k}{y}|z| = 0$ where z is positive

Assume $z = Ce^{\alpha t}$, C and α are constants

$$\dot{z} = C\alpha e^{\alpha t}$$

$$\dot{z} + \dot{d}\frac{k}{y}|z| = C\alpha e^{\alpha t} + \dot{d}\frac{k}{y}Ce^{\alpha t} = 0$$

$$\alpha + \dot{d}\frac{k}{y} = 0$$

$$\alpha = -\dot{d}\frac{k}{y}$$

Thus the homogeneous solution when z is positive is

$$z = Ce^{-\dot{d}\frac{k}{y}t}$$

Now find a particular solution for

$\dot{z} + \dot{d}\frac{K}{Y}|z| = \dot{d}\frac{K}{Y}$ where z is positive

Assume $z = A + Bt$, A and B are constants and $A \geq 0$, $B \geq 0$

$\dot{z} = B$

$$\dot{z} + \dot{d}\frac{K}{Y}|z| = B + \dot{d}\frac{K}{Y}|A + Bt| = \dot{d}\frac{K}{Y}$$

$$A\,\dot{d}\frac{K}{Y} + B\left(1 + \dot{d}\frac{K}{Y}t\right) = \dot{d}\frac{K}{Y}$$

The above equation must be true for any value of t.

for $t=0$: $A\dot{d}\frac{K}{Y} + B = \dot{d}\frac{K}{Y}$ Eqn 1

for $t = \frac{Y}{\dot{d}K}$: $A\dot{d}\frac{K}{Y} + 2B = \dot{d}\frac{K}{Y}$ Eqn 2

subtracting Eqn 1 from Eqn 2 yields

$B = 0$

Substituting $B=0$ into Eqn 1

$$A\,\dot{d}\,\frac{K}{Y} + 0 = \dot{d}\,\frac{K}{Y}$$

$$A = 1$$

Thus the particular solution is:

$$z = 1$$

Combining the homogeneous and particular solutions yields

$$\boxed{\begin{array}{l} z = 1 + Ce^{-\dot{d}\frac{K}{Y}t}, \quad \text{Eqn } 3 \\ \text{when } z > 0 \text{ and } \dot{d}z > 0 \end{array}}$$

The force-deformation results for NLSTAT1 can be calculated using equation 3. The constant C is determined by noting that when t = 0, z = 0 and substituting those values into Eqn 3.

$$0 = 1 + Ce^{0} = 1 + C$$

$$C = -1$$

$$z = 1 - e^{-\frac{K}{Y}\dot{d}t}$$

For NLSTAT1 $\dot{d}t = d - d_{initial}$. Also, the value of $d_{initial}$ is zero for NLSTAT1. Thus $\dot{d}t = d$ and the final expression for z which will be used to evaluate NLSTAT1 is:

$$z = 1 - e^{-\frac{K}{Y}d}$$

Calculate f for NLSTAT1 when d=0.20 in

$$z = 1 - e^{-\frac{K}{Y}d} = 1 - e^{-\frac{100}{50} \times 0.20} = 1 - e^{-0.4}$$

$$z = 0.329680$$

$$f = rKd + (1-r)Yz$$

$$= 0.1 \times 100 \times 0.20 + (1-0.1) \times 50 \times 0.329680$$

$$= 2 + 14.8356$$

$f = 16.8356$ K when $d = 0.20$ in for NLSTAT1

Calculate f for NLSTAT1 when d= 0.80 in

$$z = 1 - e^{-\frac{K}{Y}d} = 1 - e^{-\frac{100}{50} \times 0.80} = 1 - e^{-1.6}$$

$$z = 0.798103$$

432

$$f = rKd + (1-r)Yz$$

$$= 0.1 \times 100 \times 0.80 + (1-0.1) \times 50 \times 0.798103$$

$$= 8 + 35.9146$$

$f = 43.9146$ K when $d = 0.80$ in for NLSTAT1

Calculate f for NLSTAT1 when $d = 9.60$ in

$$z = 1 - e^{-\frac{k}{y}d} = 1 - e^{-\frac{100}{50}\times 9.60} = 1 - e^{-19.2}$$

$$z = 1.000000$$

$$f = rkd + (1-r)yz$$

$$= 0.1 \times 100 \times 9.6 + (1-0.1)\times 50 \times 1$$

$$= 96 + 45$$

$f = 141^{k}$ when $d = 9.60$ for NLSTAT1

Now find an expression for z when $\dot{d}z \le 0$

$$\dot{z} = \dot{d}\frac{k}{y}$$

$$\dot{z} - \dot{d}\frac{k}{y} = 0$$

Assume $z = A + Bt$, A and B constants

$$\dot{z} = B$$

$$\dot{z} - \dot{d}\frac{k}{y} = B - \dot{d}\frac{k}{y} = 0$$

$$B = \dot{d}\frac{k}{y}$$

$$z = A + \frac{k}{y}\dot{d}t \qquad \text{Eqn 4}$$

For Eqn 4 $\dot{d}t = d - d_{initial}$. Also, A can be determined by noting that when $t = 0$, $z = 1$ for NLSTAT2

$$1 = A + 0$$

$$A = 1$$

Thus the first portion of NLSTAT2 is calculated as

$$z = 1 + \frac{k}{y}(d - d_{initial})$$

Calculate f for NLSTAT2 when d = 9.60 in

$d_{initial} = 10$ in

$$Z = 1 + \frac{k}{y}(d - d_{initial}) = 1 + \frac{100}{50}(9.60 - 10)$$

$$Z = 0.2$$

$$f = rkd + (1-r)yz$$
$$= 0.1 \times 100 \times 9.6 + (1-0.1) \times 50 \times 0.2$$
$$= 96 + 9$$

$f = 105^{k}$ when d = 9.60 in for NLSTAT2

Find an expression for $\dot{z}$ when $\dot{d}z > 0$ and z is negative

Find homogeneous solution for $\dot{z} + \dot{d}\frac{k}{y}|z| = 0$ where z is negative

Assume $z = -Ce^{\alpha t}$, C and α constants

$$\dot{z} = -C\alpha e^{\alpha t}$$

$$\dot{z} + \dot{d}\frac{k}{y}|z| = -C\alpha e^{\alpha t} + \dot{d}\frac{k}{y}Ce^{\alpha t} = 0$$

$$-\alpha + \dot{d}\frac{k}{y} = 0$$

$$\alpha = \dot{d}\frac{k}{y}$$

Thus the homogeneous solution when z is negative is

$$z = -Ce^{\frac{k}{y}\dot{d}t}$$

Now find a particular solution for $\dot{z} + \dot{d}\frac{k}{y}|z| = \dot{d}\frac{k}{y}$ where z is negative

Assume $z = -A - Bt$, A and B are constants

$$\dot{z} = -B$$

$$\dot{z} + \dot{d}\frac{K}{y}|z| = -B + \dot{d}\frac{K}{y}(A + Bt) = \dot{d}\frac{K}{y}$$

$$A\dot{d}\frac{K}{y} + B\left(\frac{K}{y}\dot{d}t - 1\right) = \dot{d}\frac{K}{y}$$

The above expression must be valid for all t

for $t = 0$ $\quad A\dot{d}\frac{K}{y} - B = \dot{d}\frac{K}{y}$

for $t = \frac{y}{K\dot{d}}$ $\quad A\dot{d}\frac{K}{y} + 0 = \dot{d}\frac{K}{y}$

$A = 1$

$B = 0$

Thus the particular solution is:

$z = -1$

Combining the homogeneous and particular solutions yields

$$\boxed{\begin{array}{ll} z = -1 - Ce^{\frac{K}{y}\dot{d}t} & \\ \text{when } z < 0 \text{ and } \dot{d}z > 0 & \text{Eqn 5} \end{array}}$$

The force-deformation results for the portion of NLSTAT2 where $z < 0$ can be calculated using Equation 5. The constant c is evaluated by noting that when $t = 0$, $z = 0$ and substituting those values into Eqn 5.

$$0 = -1 - ce^{0} = -1 - c$$

$c = -1$, thus

$$z = -1 + e^{\frac{K}{4} \dot{d}t}$$

For NLSTAT2 $\dot{d}t = d - d_{initial}$ and $d_{initial} = 9.50$ in. Thus the final expression for z which will be used to evaluate the portion of NLSTAT2 where $z < 0$ is:

$$z = -1 + e^{\frac{K}{4}(d - 9.50)}$$

Calculate f for NLSTAT2 when d=8.60 in

$$z=-1+e^{\frac{k}{y}(d-9.50)}=-1+e^{\frac{100}{50}(8.60-9.50)}$$

$$z=-1+e^{-1.8}=-0.834701$$

$$f=rkd+(1-r)yz$$

$$=0.1\times100\times8.60+(1-0.1)\times50\times-0.834701$$

$$=86-37.5615$$

$$f=48.4384^{k} \text{ when } d=8.60'' \text{ for NLSTAT2}$$

Calculate f for NLSTAT2 when d=-9.60 in

$$z=-1+e^{\frac{k}{y}(d-9.50)}=-1+e^{\frac{100}{50}(-9.60-9.50)}$$

$$z=-1+e^{-38.2}=-1.000000$$

$$f=rkd+(1-r)yz$$

$$=0.1\times100\times-9.60+(1-0.1)\times50\times-1$$

$$=-96-45$$

$$f=-141^{k} \text{ when } d=-9.60 \text{ in for NLSTAT2}$$