算例 5-009

实体单元 – 施加在实体元对象的预应力

问题描述

SAP2000 实体元的预应力施加在这个例题中使用具有抛物线形筋两端不同偏心的简支混凝土梁进行验证的。梁中部的变形、底部和顶部的应力值程序计算结果与独立手算结果进行了对比

SAP2000 在施加预应力时有两种可以选择的方法。一种方法是在结构上直接施加预应力荷载。另外一种方法是在模型上添加预应力筋。这两种方法在本例中都进行了验证。

SAP2000 中梁是使用剖分为 2x4x10 实体对象进行的。因此，每个实体对象是 7.5-inches 高、9-inches 宽、36-inches 长。This yields an aspect ratio of 4.8。

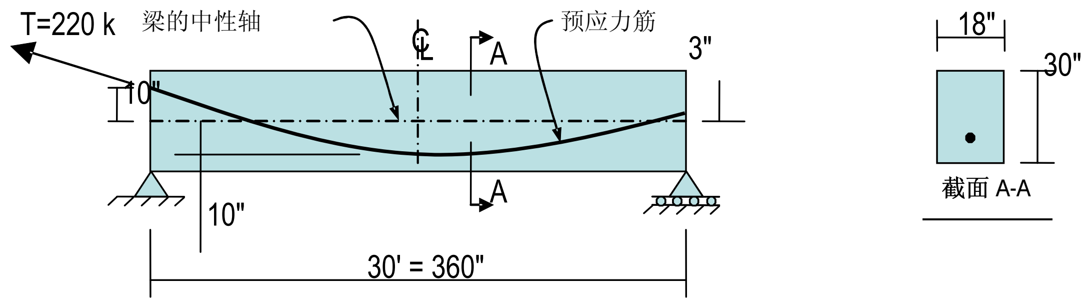
本例的分析中使用了两个独立的分析模型。模型 A 中预应力作为荷载出现，模型 B 中预应力被作为一个单元。

颤动和弯曲摩擦损失在这个例题中混凝土梁弹性压缩过程中，都进行了考虑。筋的应力只加在左端。

重要提示: 这个例子中使用了两个独立的实体元模型选项。

重要提示: 这个例子中包含了剪切变形。

几何和加载



材料属性	截面属性	预应力属性
$E = 3600\ k/in^2$	$b = 18$ in	$A = 1.5$ in2
$v = 0.2$	$d = 30$ in	$E = 29000$ k/in2
$G = 1500\ k/in^2$	$A = 540\ in^2$	$v = 0.3$
	$I = 40{,}500\ in^4$	
	$A_v = 450\ in^2$ (剪切面积)	

预应力筋注释

1. T 是在损失之前预应力筋中的拉力分量。
2. 筋只是在左端受到拉力。
3. 索在左端垂下，中间和右边在图中进行了说明。
4. 索的形状是抛物线。
5. 弯曲摩擦损失系数为 0.15。
6. 颤动摩擦损失系数为 0.0001/in。
7. 考虑梁弹性压缩和摩擦损失。
8. 设置最大筋分布长度为 12in。

所测试的**SAP2000** 技术要点：

- 具有抛物线形状和两端不同的偏心的预应力筋。
- 使用荷载的预应力筋和实体元中的使用。
- 作为单元的预应力筋和实体元中的使用。
- 预应力损失。

结果比较

手算独立结果是使用 Cook and Young 1985 第 244 页所描述的单位荷载理论方法进行计算的。

重要提示: 手算过程中一个重要的假设是分布在梁上的预应力荷载是均匀的。因此 SAP2000 和手算结果并没有期待完全一样的结果。

模型	输出参数	**SAP2000**	独立结果	误差
A- 通过荷载	平均 U_z (上部中心) in	0.16447	0.16564	-0.7%
B- 通过单元		0.16073		-3.0%
A- 通过荷载	平均 S_{11} (上部中心) kip/in^2	0.3620	0.3681	-1.7%
B- 通过单元		0.3518		-4.4%
A- 通过荷载	平均 S_{11} (上部中心) kip/in^2	-1.1216	-1.1163	+0.5%
B- 通过单元		-1.0915		-2.2%

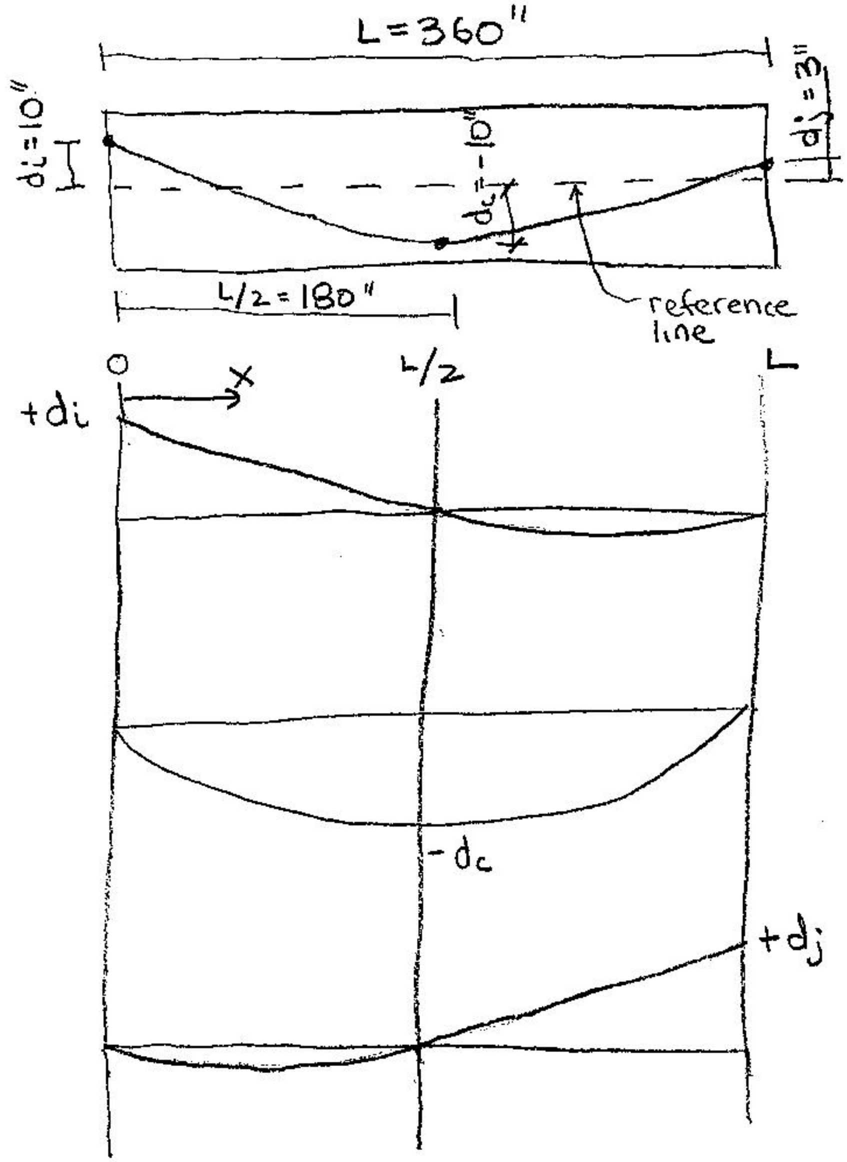
计算模型文件: Example 5-009a, Example 5-009b

结论

The SAP2000 与独立计算结果显示了可以接受的误差。

手算过程

1. Use parabolic shape functions to derive equations for tendon drape and slope.



$d(x)$ = cable drape measured positive upward from reference line

$$S(x) = \text{cable slope} = \frac{d}{dx} d(x)$$

$$d(x) = d_i \left(x - \frac{L}{2}\right)(x - L)\frac{2}{L^2} - d_c (x - 0)(x - L)\frac{4}{L^2} + d_j (x - 0)\left(x - \frac{L}{2}\right)\frac{2}{L^2}$$

$$d(x) = d_i \left(x^2 - \frac{3L}{2}x + \frac{L^2}{2}\right)\frac{2}{L^2} - d_c \left(x^2 - Lx\right)\frac{4}{L^2} + d_j \left(x^2 - \frac{Lx}{2}\right)\frac{2}{L^2}$$

$$d(x) = \left(\frac{2d_i - 4d_c + 2d_j}{L^2}\right)x^2 - \left(\frac{3d_i - 4d_c + d_j}{L}\right)x + d_i$$

$$S(x) = \frac{d}{dx} d(x) = \left(\frac{4d_i - 8d_c + 4d_j}{L^2}\right)x - \left(\frac{3d_i - 4d_c + d_j}{L}\right)$$

$$S(0) = -\left(\frac{3d_i - 4d_c + d_j}{L}\right)$$

$$S(L) = \frac{4d_i - 8d_c + 4d_j - 3d_i + 4d_c - d_j}{L} = \frac{d_i - 4d_c + 3d_j}{L}$$

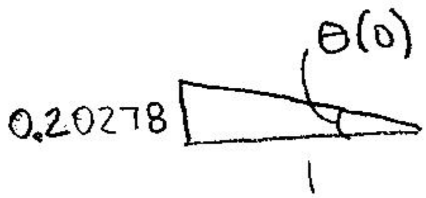
2. Calculate end slopes

$$S(0) = -\left(\frac{3\times 10 - 4\times(-10)+3}{360}\right) = -0.20278 \text{ in/in}$$

$$S(L) = \frac{10 - 4\times(-10)+3\times 3}{360} = 0.16389 \text{ in/in}$$

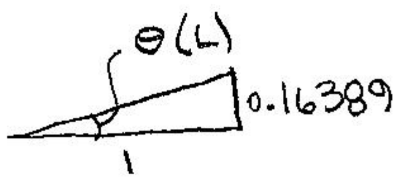
3. Calculate end angles

At L = 0":



$$\theta(0) = \text{Atan}(0.20278)$$
$$\theta(0) = 11.463°$$

At L = 360":



$$\theta(L) = \text{Atan}(0.16389)$$
$$\theta(L) = 9.307°$$

4. Determine friction loss coefficients at $L = 360''$

$$\text{Curvature Loss Coefficient} = \frac{0.15 \times (11.463^\circ + 9.307^\circ) \times \pi}{180}$$

$$= 0.05438$$

$$\text{Wobble Loss Coefficient} = 0.0001 \times 360'' = 0.036$$

5. Determine axial end forces before elastic shortening occurs

At $L = 0''$: $F_A = 220 \cos(11.463^\circ)$

$F_A = 215.612^k$

At $L = 360''$:

$F_A = 220(1 - 0.05438 - 0.036)\cos(9.307^\circ)$

$F_A = 197.482^k$

6. Estimate tension loss from axial shortening

P = average axial force

$$P = \frac{215.612 + 197.482}{2} = 206.5^k$$

$$\Delta_{conc} = \frac{PL}{E_{conc} A_{conc}}$$

$$\Delta_{conc} = \frac{206.5 \times 360}{3600 \times 18 \times 30} = 0.0382 \text{ in}$$

$$P_{tendon} = \frac{E_{tendon} A_{tendon} \Delta_{conc}}{L}$$

$$P_{tendon} = \frac{29000 \times 1.5 \times 0.0382}{360} = 4.62^k$$

Assume 4.6^k loss from axial shortening

$$\text{Stress loss} = \frac{4.6}{1.5} = 3.067 \text{ Ksi}$$

This loss is specified in Model A where the prestress is represented as loads.

7. Calculate end forces at $L = 0''$

$$F_A = (220 - 4.6)\cos(11.463^\circ) = 211.103^k \rightarrow$$

$$F_V = (220 - 4.6)\sin(11.463^\circ) = 42.808^k \downarrow$$

$$M = 10 F_A = 211.103 \times 10 = 2111.03 \text{ k-in}$$

8. Calculate end forces at $L = 360''$

$$F_A = [\{220(1 - 0.05438 - 0.036)\} - 4.6]\cos(9.307^\circ)$$

$$F_A = 192.943^k \leftarrow$$

$$F_V = [\{220(1 - 0.05438 - 0.036)\} - 4.6]\sin(9.307^\circ)$$

$$F_V = 31.620^k \downarrow$$

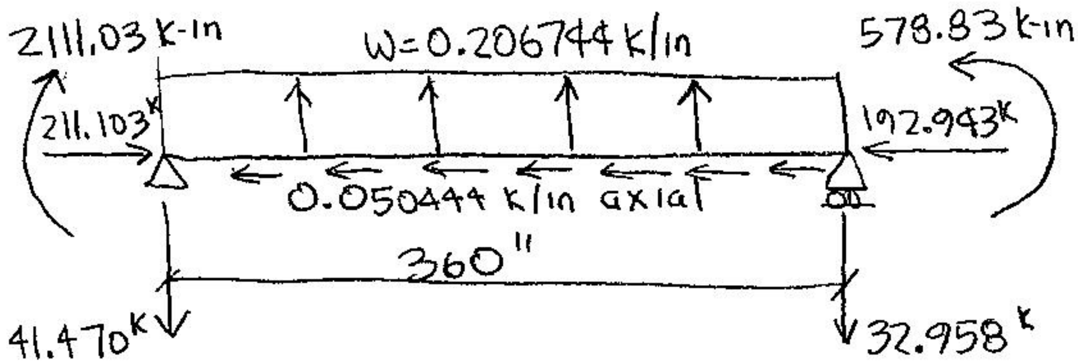
$$M = 3 F_A = 3 \times 192.943 = 578.83 \text{ k-in}$$

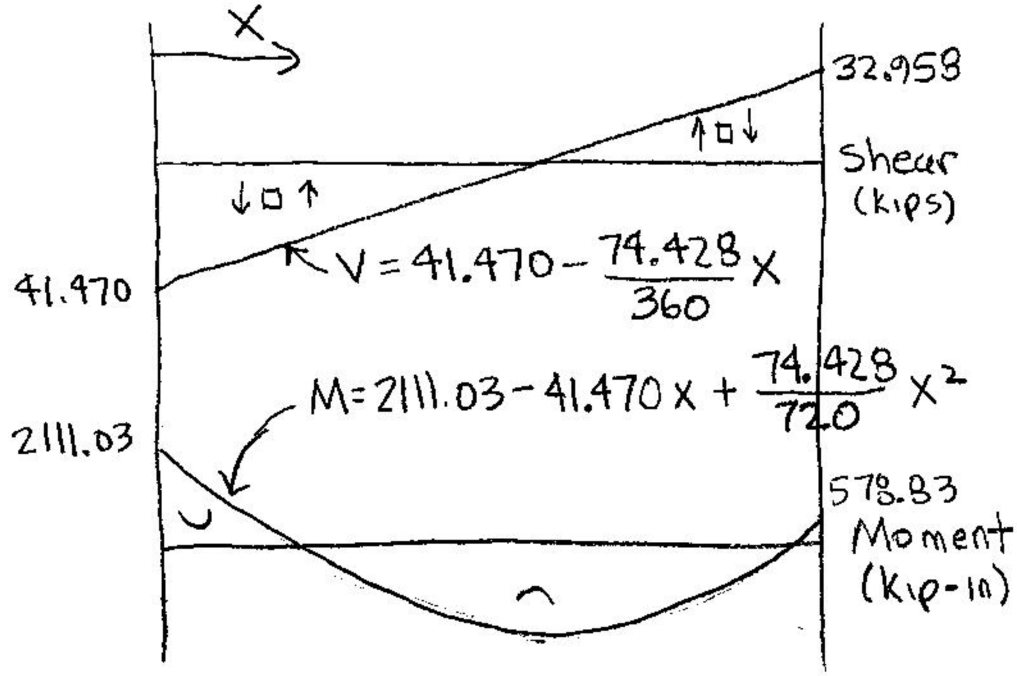
9. Determine magnitude of upward distributed load due to prestress. For simplicity in this hand calculation assume the distributed load is uniform.

$$w = \frac{(F_V @ L = 0) + (F_V @ L = 360)}{L} = \frac{42.808 + 31.620}{360}$$

$$w = 0.206744 \text{ K/in} \uparrow$$

10. Plot equivalent loading on beam. Note that assumed uniform upward load from prestress yields calculated vertical reactions that are slightly different from previously calculated end forces





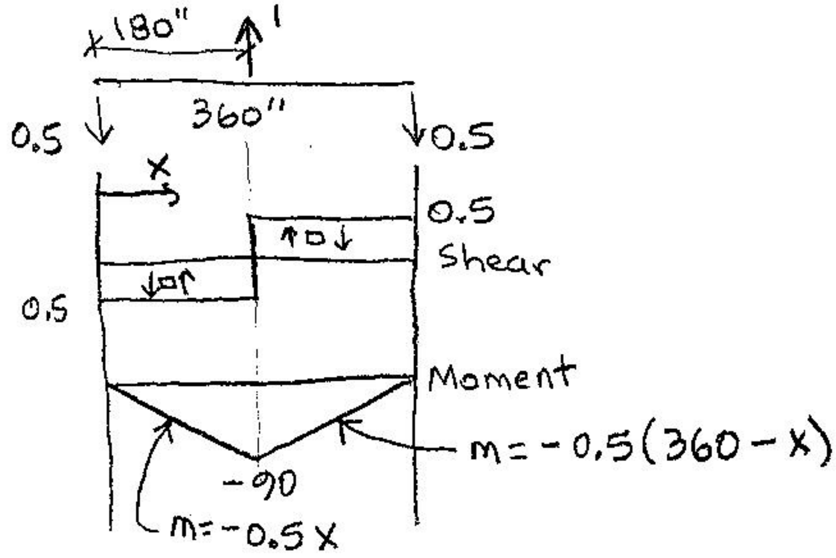
11. Calculate midspan deflection caused by prestress alone considering flexural and shear deformations.

Use $\Delta = \int \frac{mM}{EI} dx + \int \frac{vV}{GA_v} dx$

$$EI = \frac{3600 \times 18 \times 30^3}{12} = 145{,}800{,}000$$

$$GA_v = 1500 \times \frac{5}{6} \times 18 \times 30 = 675{,}000$$

Virtual System:



Flexural Deformation:

$$\Delta_f = \frac{1}{EI}\int_0^{180}\left(-1055.515x + 20.735x^2 - \frac{74.428}{1440}x^3\right)dx$$

$$+\frac{1}{EI}\int_{180}^{360}\left(-379985.4 + 7464.6x - \frac{74.428}{4}x^2\right)dx$$

$$-\frac{1}{EI}\int_{180}^{360}\left(-1055.515x + 20.735x^2 - \frac{74.428}{1440}x^3\right)dx$$

$$\Delta_f = \frac{1}{EI}\left(\frac{-1055.515x^2}{2} + \frac{20.735x^3}{3} - \frac{74.428x^4}{5760}\right)\Bigg|_0^{180}$$

$$+\frac{1}{EI}\left(-379985.4x + \frac{7464.6x}{2} - \frac{74.428x^3}{12}\right)\Bigg|_{180}^{360}$$

$$-\frac{1}{EI}\left(\frac{-1055.515x^2}{2} + \frac{20.735x^3}{3} - \frac{74.428x^4}{5760}\right)\Bigg|_{180}^{360}$$

$$\Delta_f = -0.117279 + 0.276467 - 0.093035$$
$$-0.938236 + 3.3176 - 1.984747$$
$$+0.469118 - 0.8294 + 0.248093$$
$$+0.469118 - 2.211733 + 1.48856$$
$$-0.117279 + 0.276467 - 0.093035$$

$\Delta_f = 0.160679$ in $\uparrow$, flexural deformation

Shear Deformation:

$$\Delta_v = \frac{1}{GA_v}\int_0^{180}\left(\frac{41.47}{2} - \frac{74.428}{720}x\right)dx$$
$$+ \frac{1}{GA_v}\int_{180}^{360}\left(-\frac{41.47}{2} + \frac{74.428}{720}x\right)dx$$

$$\Delta_v = \frac{1}{GA_v}\left(\frac{41.47x}{2} - \frac{74.428x^2}{1440}\right)\Bigg|_0^{180}$$
$$+ \frac{1}{GA_v}\left(-\frac{41.47x}{2} + \frac{74.428x^2}{1440}\right)\Bigg|_{180}^{360}$$

$$\Delta_v = 0.005529 - 0.002481$$
$$-0.011059 + 0.009924$$
$$+0.005529 - 0.002481$$

$\Delta_v = 0.004961$ in ↑, shear deformation

Total Deformation:

$$\Delta_{total} = \Delta_f + \Delta_v = 0.160679 + 0.004961$$

$$\underline{\underline{\Delta_{total} = 0.1656 \text{ in } \uparrow}}$$

12. Calculate center moment caused by prestress alone

$$M_{center} = 2111.03 - 41.47 \times 180 + \frac{74.428}{720} \times 180^2$$

$$M_{center} = -2004.3 \text{ K-in}$$

13. Calculate top and bottom S11 stress at midspan caused by prestress alone.

$$P_{center} = \frac{-211.103-192.943}{2} = -202.023^{K}$$
compression

$$\frac{P}{A} = \frac{P_{center}}{A_{conc}} = \frac{-202.023}{18\times30} = -0.3741 \text{ ksi}$$

$$I = \frac{bd^3}{12} = \frac{18\times30^3}{12} = 40{,}500 \text{ in}^4$$

$$\frac{Mc}{I} = \frac{-2004.3\times15}{40500} = -0.7422 \text{ ksi}$$

Top Stress:

$$\frac{P}{A} - \frac{Mc}{I} = -0.3741+0.7422 = 0.3681 \text{ ksi}$$
tension

Bottom Stress:

$$\frac{P}{A} + \frac{Mc}{I} = -0.3741-0.7422 = -1.1163 \text{ ksi}$$
compression