算例 2-004

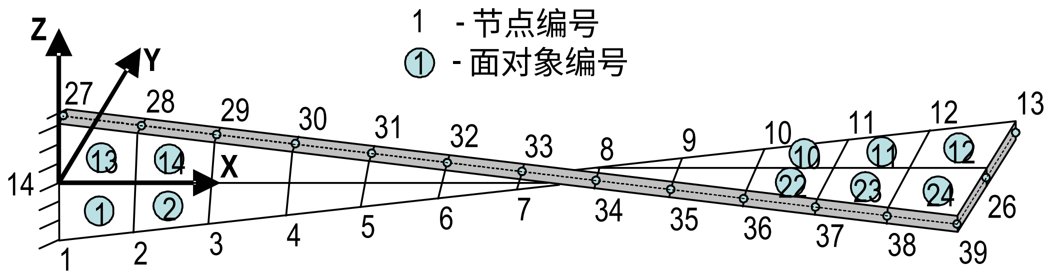
壳 – 静载下的扭曲梁

算例 描述

本例的模型为一个由壳元模拟的扭曲的悬臂梁，在端部承受面内和面外（Y 和 Z 向）单位力。面内和面外力的工况不同。将端部面内和面外的外力方向产生的位移与手算结果进行了对比。

MacNeal and Harder 1985 一书给出了模型的几何特性、属性和荷载。该悬臂梁从根部（固接端）到端部扭转了 90 度。该梁长 12in.，宽 1.1in.，高 0.32in.。为了进行建模，SAP2000 中将该梁剖分为 2 乘 12 个（共 24 个面对象）单元，如图所示。

几何特性和属性



几何属性	材料属性	截面属性
长度 = 12.0 in	E = 29,000,000 lb/in^2	厚度 = 0.32 in
宽度 =1.1 in	v = 0.22	
转角= 90° (根部到顶部)	G = 11,885,246 lb/in^2	

荷载

下表列出了在各模型中施加的面内和面外荷载。

荷载工况	荷载
面内	F_y = +0.25 lb 节点 13 处 F_y = +0. 50 lb 节点 26 处 F_y = +0.25 lb 节点 39 处
面外	F_z = +0.25 lb 节点 13 处 F_z = +0. 50 lb 节点 26 处 F_z = +0.25 lb 节点 39 处

所测试的 SAP2000 技术要点：

- 采用壳元时的膜分析
- 采用壳元时的板分析
- 节点荷载施加

结果比较

分别列出了采用薄板选项和厚板选项时的 SAP2000 的结果。采用 Cook and Young 1985 一书第 244 页的单位力法手算得出手算解。MacNeal and Harder 1985 一书中也给出了手算解。

采用薄板选项时

模型和剖分	荷载工况和类型	输出参数	SAP2000	手算解	差值百分比
薄板 12x2 网格	荷载工况 IN 面内	U_y, in 13, 26, 39 节点平均值	0.005415	0.005429	-0.3%
	荷载工况 OUT 面外	U_z, in 13, 26, 39 节点平均值	0.001769	0.001749	+1.1%

采用厚板选项时

模型和剖分	荷载工况和类型	输出参数	SAP2000	手算解	差值百分比
厚板 12x2 网格	荷载工况 IN 面内	U_y, in 13, 26, 39 节点平均值	0.005402	0.005429	-0.5%
	荷载工况 OUT 面外	U_z, in 13, 26, 39 节点平均值	0.001760	0.001749	0.6%

计算模型文件: Example 2-004-thick, Example 2-004-thin

结论

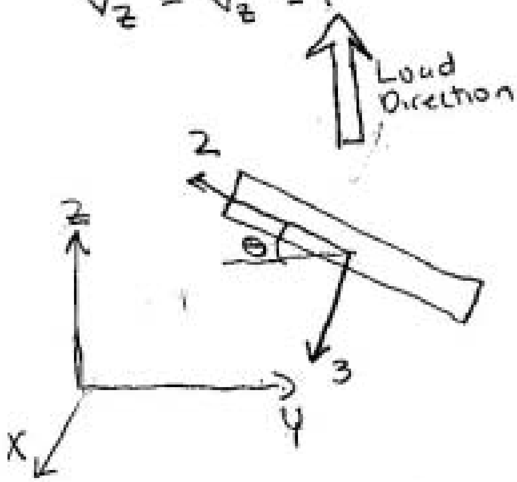
SAP2000 与手算解的差异是可以接受的。一般而言，进一步剖分模型时，结果间的差异会更小。

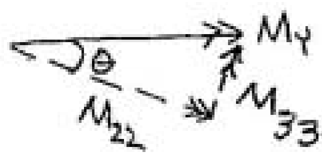
手算过程

Twisted Beam with Out-of-Plane loading at Tip

Unit Load in Z direction

$M_y = m_y = S$ where M_y = actual moment
m_y = virtual moment
S = distance along beam measured from tip
$V_z = v_z = 1$
V_z = actual shear
v_z = virtual shear





$$M_{22} = M_y \cos\theta = S\cos\theta$$

$$M_{33} = M_y \sin\theta = S\sin\theta$$

$$\theta = \frac{\pi}{2L} S$$

$$V_Z = v_z = 1$$

$$V_2 = V_z \sin\theta = \sin\theta$$

$$V_3 = V_z \cos\theta = \cos\theta \rightarrow$$

$$\Delta = \int_0^L \frac{m_{22} M_{22}}{EI_{22}} ds + \int_0^L \frac{m_{33} M_{33}}{EI_{33}} ds + \int_0^L \frac{v_2 V_2}{GA_v} ds + \int_0^L \frac{v_3 V_3}{GA_v} ds$$

$$\Delta = \int_0^L \frac{s^2 \cos^2\left(\frac{\pi}{2L} s\right)}{EI_{22}} ds + \int_0^L \frac{s^2 \sin^2\left(\frac{\pi}{2L} s\right)}{EI_{33}} ds + \int_0^L \frac{\sin^2\left(\frac{\pi}{2L} s\right)}{GA_v} ds + \int_0^L \frac{\cos^2\left(\frac{\pi}{2L} s\right)}{GA_v} ds$$

From integral tables

$$\Delta = \frac{1}{EI_{22}}\left[\frac{s^3}{6} + \left(\frac{Ls^2}{2\pi} - \frac{L^3}{\pi^3}\right)\sin\left(\frac{\pi}{L}s\right) + \frac{L^2 s}{\pi^2}\cos\left(\frac{\pi}{L}s\right)\right]\Bigg|_0^L$$

$$+ \frac{1}{EI_{33}}\left[\frac{s^3}{6} - \left(\frac{Ls^2}{2\pi} - \frac{L^3}{\pi^3}\right)\sin\left(\frac{\pi}{L}s\right) - \frac{L^2 s}{\pi^2}\cos\left(\frac{\pi}{L}s\right)\right]\Bigg|_0^L$$

$$+ \frac{1}{GA_v}\left[\frac{s}{2} - \frac{L}{2\pi}\sin\left(\frac{\pi}{L}s\right)\right]\Bigg|_0^L + \frac{1}{GA_v}\left[\frac{s}{2} + \frac{L}{2\pi}\sin\left(\frac{\pi}{L}s\right)\right]\Bigg|_0^L$$

$$\Delta = \frac{1}{EI_{22}}\left[\frac{L^3}{6} - \frac{L^3}{\pi^2}\right] + \frac{1}{EI_{33}}\left[\frac{L^3}{6} + \frac{L^3}{\pi^2}\right] + \frac{2}{GA_v}\left[\frac{L}{2}\right]$$

$$\Delta = \frac{(\pi^2 - 6)L^3}{6\pi^2 EI_{22}} + \frac{(\pi^2 + 6)L^3}{6\pi^2 EI_{33}} + \frac{L}{GA_v}$$

$$\Delta = \frac{(\pi^2 - 6)(12^3)}{6\pi^2 \times 29000000 \times \left(\frac{1.1 \times 0.32^3}{12}\right)}$$

$$+ \frac{(\pi^2 + 6) \times 12^3}{6\pi^2 \times 29000000 \times \left(\frac{0.32 \times 1.1^3}{12}\right)}$$

$$+ \frac{12}{11885296 \times \left(\frac{5}{6} \times 1.1 \times 0.32\right)}$$

$$\Delta = 0.00129628 + 0.00044990 + 0.00000344$$

$$\boxed{\Delta = 0.0017487 \text{ in} \quad \text{for out-of-plane load at tip}}$$

Twisted Beam with In-Plane Loading at tip

Unit load in Y direction

$M_z = m_z = S$ where M_z = actual moment

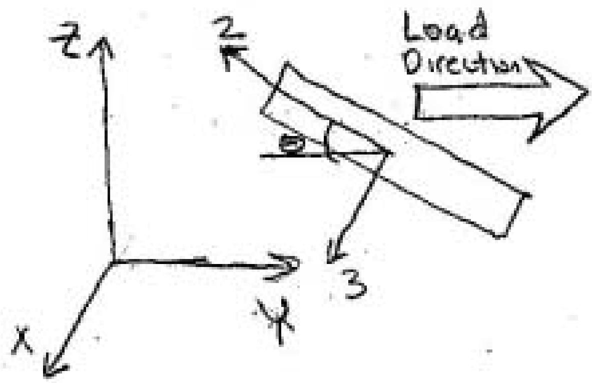
m_z = virtual moment

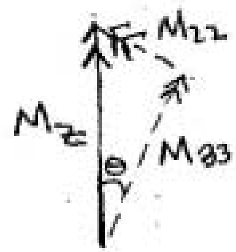
S = distance along beam measured from tip

$V_y = v_y = 1$

V_y = actual shear

v_y = virtual shear



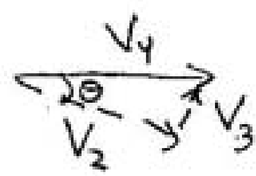


$M_{22} = M_z \sin\theta = S\sin\theta$

$M_{33} = M_z \cos\theta = S\cos\theta$

$\theta = \frac{\pi}{2L} S$

$V_y = v_y = 1$



$V_2 = V_y \cos\theta = \cos\theta$

$V_3 = V_y \sin\theta = \sin\theta$

$$\Delta = \int_0^L \frac{m_{22} M_{22}}{EI_{22}} ds + \int_0^L \frac{m_{33} M_{33}}{EI_{33}} ds + \int_0^L \frac{v_2 V_2}{GA_v} ds + \int_0^L \frac{v_3 V_3}{GA_v}$$

$$\Delta = \int_0^L \frac{S^2 \sin^2\left(\frac{\pi}{2L} s\right)}{EI_{22}} ds + \int_0^L \frac{S^2 \cos^2\left(\frac{\pi}{2L} s\right)}{EI_{33}} ds + \int_0^L \frac{\cos^2\left(\frac{\pi}{2L} s\right)}{GA_v} ds + \int_0^L \frac{\sin^2\left(\frac{\pi}{2L} s\right)}{GA_v} ds$$

From integral tables

$$\Delta = \frac{1}{EI_{22}} \left[\frac{S^3}{6} - \left(\frac{LS^2}{2\pi} - \frac{L^3}{\pi^3} \right) \sin\left(\frac{\pi}{L} s\right) - \frac{L^2 s}{\pi^2} \cos\left(\frac{\pi}{L} s\right) \right] \Bigg|_0^L$$

$$+ \frac{1}{EI_{33}} \left[\frac{S^3}{6} + \left(\frac{LS^2}{2\pi} - \frac{L^3}{\pi^3} \right) \sin\left(\frac{\pi}{L} s\right) + \frac{L^2 s}{\pi^2} \cos\left(\frac{\pi}{L} s\right) \right] \Bigg|_0^L$$

$$+ \frac{1}{GA_v} \left[\frac{S}{2} + \frac{L}{2\pi} \sin\left(\frac{\pi}{L} s\right) \right]_0^L + \frac{1}{GA_v} \left[\frac{S}{2} - \frac{L}{2\pi} \sin\left(\frac{\pi}{L} s\right) \right] \Bigg|_0^L$$

$$\Delta = \frac{1}{EI_{22}} \left[\frac{L^3}{6} + \frac{L^3}{\pi^2} \right] + \frac{1}{EI_{33}} \left[\frac{L^3}{6} - \frac{L^3}{\pi^2} \right] + \frac{2}{GA_v} \left[\frac{L}{2} \right]$$

$$\Delta = \frac{(\pi^2 + 6) L^3}{6\pi^2 EI_{22}} + \frac{(\pi^2 - 6) L^3}{6\pi^2 EI_{33}} + \frac{L}{GA_v}$$

$$\Delta = \frac{(\pi^2+6)(12^3)}{6\pi^2 \times 29000000 \times \left(\frac{1.1 \times 0.32^3}{12}\right)}$$

$$+ \frac{(\pi^2-6)(12^3)}{6\pi^2 \times 29000000 \times \left(\frac{0.32 \times 1.1^3}{12}\right)}$$

$$+ \frac{12}{11885246 \times \left(\frac{5}{6} \times 1.1 \times 0.32\right)}$$

$$\Delta = 0.00531618 + 0.00010970 + 0.00000344$$

$$\boxed{\Delta = 0.00542932 \text{ in} \quad \text{for in-plane load at tip}}$$