算例 5-008

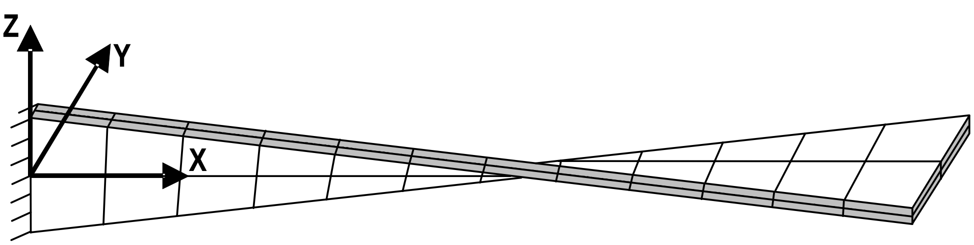
实体单元 –静载下的扭曲梁

问题描述

本例的模型为一个由实体元模拟的扭曲的悬臂梁，在端部承受面内和面外（Y 和 Z 向）单位力。面内和面外力的工况不同。将端部面内和面外的外力方向产生的位移与手算结果进行了对比。

MacNeal and Harder 1985 一书给出了模型的几何特性、属性和荷载。该悬臂梁从根部（固接端）到端部扭转了 90 度。该梁长 12-in.，宽 1.1-in.，高 0.32-in.。为了进行建模，SAP2000 中将该梁剖分为 12 x 2 x 2 个（共 48 个实体对象）单元，如图所示。

几何特性和属性



几何属性

长度 = 12.0 in
宽度 =1.1 in
转角= 90° (根部到顶部)
剖分＝12X2X2

材料属性

E = 29,000,000 lb/in^2
v = 0.22
G = 11,885,246 lb/in^2

截面属性

厚度 = 0.32 in

荷载

下表列出了在各模型中施加的面内和面外荷载。

荷载工况	荷载
面内	F_y = +1 lb 在所有梁自由端部节点均匀分布
面外	F_z = +1 lb 在所有梁自由端部节点均匀分布

所测试的 SAP2000 技术要点：

- 独立弯曲模型选项的实体对象的扭曲
- 节点荷载施加

结果比较

本例中采用 Cook and Young 1985 一书第 244 页的单位力法手算得出手算解。MacNeal and Harder 1985 一书中也给出了手算解。

独立的弯曲模型

荷载工况和类型	输出参数	SAP2000	手算解	误差
板面内荷载工况	U_y, in 顶部所有节点的平均值	0.005322	0.005429	-2.0%
板面外荷载工况	U_z, in 顶部所有节点的平均值	0.001719	0.001749	-1.7%

计算模型文件: Example 5-004

结论

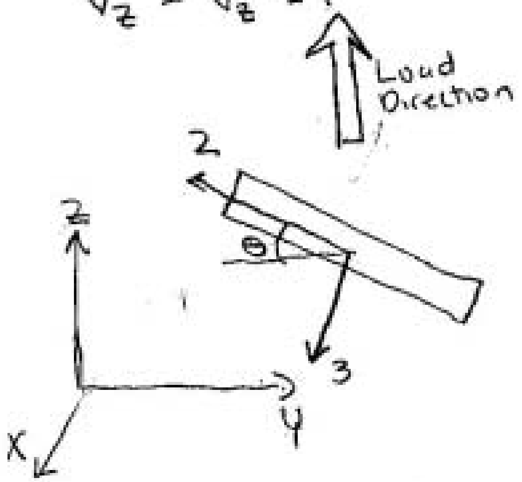
SAP2000 与手算解的差异是可以接受的。一般而言，进一步剖分模型时，结果间的差异会更小。

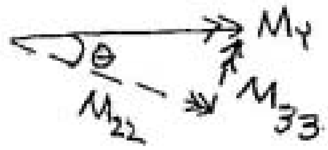
手算过程

Twisted Beam with Out-of-Plane loading at Tip

Unit Load in Z direction

$M_y = m_y = S$ where M_y = actual moment
m_y = virtual moment
S = distance along beam measured from tip
$V_z = v_z = 1$ V_z = actual shear
v_z = virtual shear





$$M_{22} = M_y \cos\theta = S\cos\theta$$

$$M_{33} = M_y \sin\theta = S\sin\theta$$

$$\theta = \frac{\pi}{2L} S$$

$$V_Z = v_z = 1$$

$V_2 = V_z \sin\theta = \sin\theta$

$V_3 = V_z \cos\theta = \cos\theta \rightarrow$

$$\Delta = \int_0^L \frac{m_{22} M_{22}}{EI_{22}} ds + \int_0^L \frac{m_{33} M_{33}}{EI_{33}} ds + \int_0^L \frac{v_2 V_2}{GA_v} ds + \int_0^L \frac{v_3 V_3}{GA_v} ds$$

$$\Delta = \int_0^L \frac{s^2 \cos^2\left(\frac{\pi}{2L} s\right) ds}{EI_{22}} + \int_0^L \frac{s^2 \sin^2\left(\frac{\pi}{2L} s\right) ds}{EI_{33}} + \int_0^L \frac{\sin^2\left(\frac{\pi}{2L} s\right)}{GA_v} ds + \int_0^L \frac{\cos^2\left(\frac{\pi}{2L} s\right)}{GA_v} ds$$

From integral tables

$$\Delta = \frac{1}{EI_{22}} \left[\frac{s^3}{6} + \left(\frac{Ls^2}{2\pi} - \frac{L^3}{\pi^3} \right) \sin\left(\frac{\pi}{L} s\right) + \frac{L^2 s}{\pi^2} \cos\left(\frac{\pi}{L} s\right) \right] \Bigg|_0^L$$

$$+ \frac{1}{EI_{33}} \left[\frac{s^3}{6} - \left(\frac{Ls^2}{2\pi} - \frac{L^3}{\pi^3} \right) \sin\left(\frac{\pi}{L} s\right) - \frac{L^2 s}{\pi^2} \cos\left(\frac{\pi}{L} s\right) \right] \Bigg|_0^L$$

$$+ \frac{1}{GA_v} \left[\frac{s}{2} - \frac{L}{2\pi} \sin\left(\frac{\pi}{L} s\right) \right] \Bigg|_0^L + \frac{1}{GA_v} \left[\frac{s}{2} + \frac{L}{2\pi} \sin\left(\frac{\pi}{L} s\right) \right] \Bigg|_0^L$$

$$\Delta = \frac{1}{EI_{22}} \left[\frac{L^3}{6} - \frac{L^3}{\pi^2} \right] + \frac{1}{EI_{33}} \left[\frac{L^3}{6} + \frac{L^3}{\pi^2} \right] + \frac{2}{GA_v} \left[\frac{L}{2} \right]$$

$$\Delta = \frac{(\pi^2 - 6) L^3}{6\pi^2 EI_{22}} + \frac{(\pi^2 + 6) L^3}{6\pi^2 EI_{33}} + \frac{L}{GA_v}$$

$$\Delta = \frac{(\pi^2-6)(12^3)}{6\pi^2 \times 29000000 \times \left(\frac{1.1 \times 0.32^3}{12}\right)}$$

$$+ \frac{(\pi^2+6)\times 12^3}{6\pi^2 \times 29000000 \times \left(\frac{0.32 \times 1.1^3}{12}\right)}$$

$$+ \frac{12}{11885296 \times \left(\frac{5}{6} \times 1.1 \times 0.32\right)}$$

$$\Delta = 0.00129628 + 0.00044990 + 0.00000344$$

$$\boxed{\Delta = 0.0017487 \text{ in} \quad \text{for out-of-plane load at tip}}$$

Twisted Beam with In-Plane Loading at tip

Unit load in Y direction

$M_z = m_z = S$ where M_z = actual moment

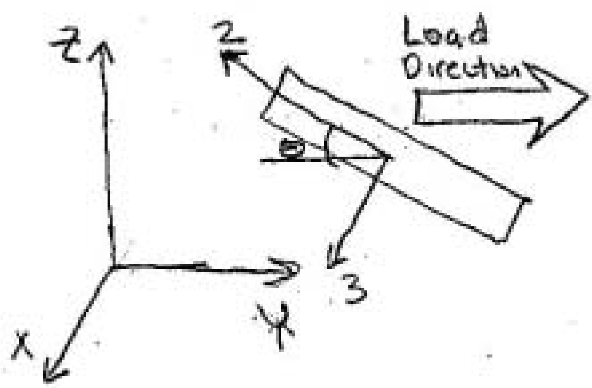
$V_y = v_y = 1$

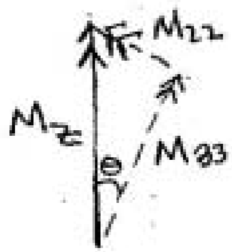
m_z = virtual moment

S = distance along beam measured from tip

V_y = actual shear

v_y = virtual shear





$$M_{22} = M_z \sin\theta = S \sin\theta$$

$$M_{33} = M_z \cos\theta = S \cos\theta$$

$$\theta = \frac{\pi}{2L} S$$

$$V_y = v_y = 1$$

$$V_2 = V_y \cos\theta = \cos\theta$$

$$V_3 = V_y \sin\theta = \sin\theta$$

$$\Delta = \int_0^L \frac{m_{22} M_{22}}{EI_{22}} ds + \int_0^L \frac{m_{33} M_{33}}{EI_{33}} ds + \int_0^L \frac{v_2 V_2}{GA_v} ds + \int_0^L \frac{v_3 V_3}{GA_v}$$

$$\Delta = \int_0^L \frac{S^2 \sin^2\left(\frac{\pi}{2L} s\right)}{EI_{22}} ds + \int_0^L \frac{S^2 \cos^2\left(\frac{\pi}{2L} s\right)}{EI_{33}} ds + \int_0^L \frac{\cos^2\left(\frac{\pi}{2L} s\right)}{GA_v} ds + \int_0^L \frac{\sin^2\left(\frac{\pi}{2L} s\right)}{GA_v} ds$$

From integral tables

$$\Delta = \frac{1}{EI_{22}} \left[\frac{S^3}{6} - \left(\frac{LS^2}{2\pi} - \frac{L^3}{\pi^3} \right) \sin\left(\frac{\pi}{L} s\right) - \frac{L^2 S}{\pi^2} \cos\left(\frac{\pi}{L} s\right) \right] \Bigg|_0^L$$

$$+ \frac{1}{EI_{33}} \left[\frac{S^3}{6} + \left(\frac{LS^2}{2\pi} - \frac{L^3}{\pi^3} \right) \sin\left(\frac{\pi}{L} s\right) + \frac{L^2 S}{\pi^2} \cos\left(\frac{\pi}{L} s\right) \right] \Bigg|_0^L$$

$$+ \frac{1}{GA_v} \left[\frac{S}{2} + \frac{L}{2\pi} \sin\left(\frac{\pi}{L} s\right) \right]_0^L + \frac{1}{GA_v} \left[\frac{S}{2} - \frac{L}{2\pi} \sin\left(\frac{\pi}{L} s\right) \right] \Bigg|_0^L$$

$$\Delta = \frac{1}{EI_{22}} \left[\frac{L^3}{6} + \frac{L^3}{\pi^2} \right] + \frac{1}{EI_{33}} \left[\frac{L^3}{6} - \frac{L^3}{\pi^2} \right] + \frac{2}{GA_v} \left[\frac{L}{2} \right]$$

$$\Delta = \frac{(\pi^2 + 6) L^3}{6\pi^2 EI_{22}} + \frac{(\pi^2 - 6) L^3}{6\pi^2 EI_{33}} + \frac{L}{GA_v}$$

$$\Delta = \frac{(\pi^2+6)(12^3)}{6\pi^2 \times 29000000 \times \left(\frac{1.1 \times 0.32^3}{12}\right)}$$

$$+ \frac{(\pi^2-6)(12^3)}{6\pi^2 \times 29000000 \times \left(\frac{0.32 \times 1.1^3}{12}\right)}$$

$$+ \frac{12}{11885246 \times \left(\frac{5}{6} \times 1.1 \times 0.32\right)}$$

$$\Delta = 0.00531618 + 0.00010970 + 0.00000344$$

$$\boxed{\Delta = 0.00542932 \text{ in} \quad \text{for in-plane load at tip}}$$