算例 2-020

桥 – 施加给面对象的预应力

算例描述

本例使用一个带抛物线预应力筋和不同端部偏心的简支混凝土梁，验证 SAP2000 的施加给面对象的预应力。将挠度以及顶部、中部和底部应力 与手算结果进行了对比。

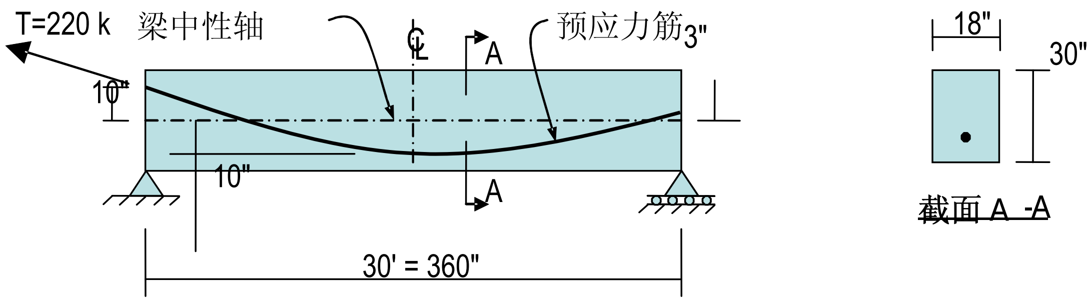
SAP2000 有两种方式模拟预应力效应。一种方式模拟施加在结构上的作为外荷载的预应力。另一种方式模拟预应力筋。本例对这两种方式都进行验证。

SAP2000 中，使用 4x10 面对象网格模拟该梁。因此，各面对象高为 7.5 英寸、长为 36 英寸。其方面比率为 4.8。

分析中使用两个不同的模型。模型 A 具有模拟为荷载的预应力。模型 B 的预应力体现在单元上。本例中包括了错动和曲线预应力损失效应，以及混凝土梁的弹性缩短。预应力筋只从左侧施加预应力。

重要提示：本例中考虑剪切变形。

几何特性和荷载



材料属性

E = 3600 k/in^2
v = 0.2
G= 1500 k/in^2

截面属性

b = 18 in
d = 30 in
A = 540 in^2
I = 40,500 in^4
A_v = 450 in^2（剪切面积）

预应力属性

A = 1.5 in2
E = 29000 k/in2
v = 0.3

预应力筋注意事项：

1. T 是预应力筋在预应力损失之前的拉力分量。
2. 筋仅从左侧张拉。
3. 图中显示了梁的左、右、中部筋距中性轴的距离。
4. 筋形状为抛物线。
5. 曲线摩擦损失系数为 0.15。
6. 错动摩擦损失系数为 0.0001 / 英寸。
7. 考虑摩擦和梁弹性缩短引起的损失。
8. 最大筋细分长度为 12 英寸。

校验的SAP2000 的技术特色

- 预应力筋的抛物线形状、在两端具有不同偏心
- 使用荷载模拟的预应力筋及其在面对象上的施加
- 使用单元模拟的预应力筋及其在面对象上的施加
- 预应力损失

结果对比

使用基本理论和 Cook and Young 1985 第 244 页的单位力法得到独立手算结果。对使用薄板和厚板选项的模型分别给出了结果。

重要提示：手工计算时，假定作用于梁上的分布荷载为均布的。因此并不期望 SAP2000 和手算结果完全相同。

薄板选项

模型	输出参数	SAP2000	独立结果	差异百分比
A- 通过荷载	U_z （中心） in	0.16465	0.16564	-0.6%
B- 通过单元		0.16094		-2.8%
A-通过荷载	S_{11} 平均值 （顶部中心） kip/in^2	0.3703	0.3681	+0.6%
B-通过单元		0.3603		-2.1%
A-通过荷载	S_{11} 平均值 (bot. center) kip/in^2	-1.1286	-1.1163	+1.1%
B-通过单元		-1.0987		-1.6%

厚板选项

模型	输出参数	SAP2000	独立结果	差异百分比
A- 通过荷载	U_z （中心） in	0.16465	0.16564	-0.6%
B- 通过单元		0.16094		-2.8%
A-通过荷载	Avg. S_{11} （顶部中心） kip/in^2	0.3703	0.3681	+0.6%
B-通过单元		0.3603		-2.1%
A-通过荷载	S_{11} 平均值 （底部中心） kip/in^2	-1.1286	-1.1163	+1.1%
B-通过单元		-1.0987		-1.6%

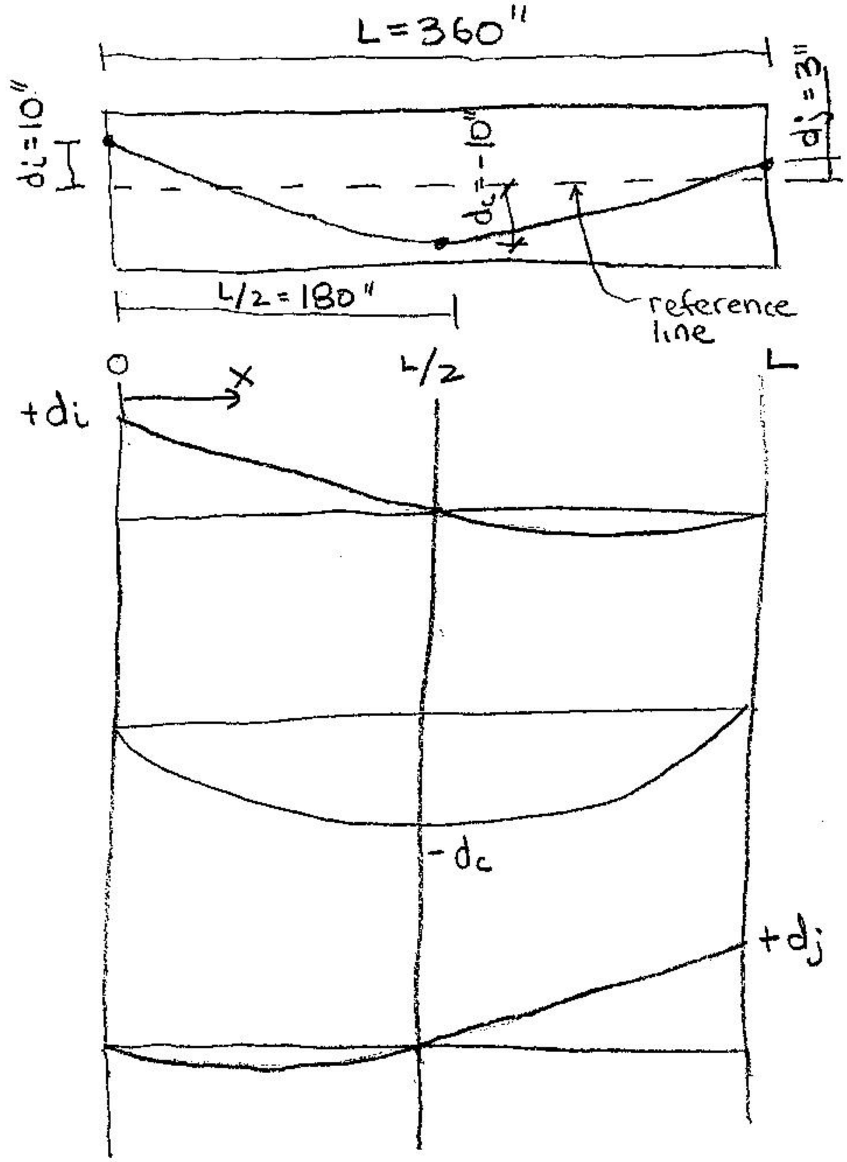
计算模型文件： Example 2-020a-thick, Example 2-020a-thin, Example 2-020b-thick, Example 2-020b-thin

结论

SAP2000 与独立结果吻合程度可以接受。

手算过程

1. Use parabolic shape functions to derive equations for tendon drape and slope.



$d(x)$ = cable drape measured positive upward from reference line

$$S(x) = \text{cable slope} = \frac{d}{dx} d(x)$$

$$d(x) = d_i \left(X - \frac{L}{2}\right)(X - L)\frac{2}{L^2} - d_c (X-0)(X-L)\frac{4}{L^2}$$

$$+ d_j (X-0)\left(X - \frac{L}{2}\right)\frac{2}{L^2}$$

$$d(x) = d_i \left(X^2 - \frac{3L}{2}X + \frac{L^2}{2}\right)\frac{2}{L^2} - d_c \left(X^2 - LX\right)\frac{4}{L^2}$$

$$+ d_j \left(X^2 - \frac{LX}{2}\right)\frac{2}{L^2}$$

$$d(x) = \left(\frac{2d_i - 4d_c + 2d_j}{L^2}\right)X^2 - \left(\frac{3d_i - 4d_c + d_j}{L}\right)X + d_i$$

$$S(x) = \frac{d}{dx} d(x) = \left(\frac{4d_i - 8d_c + 4d_j}{L^2}\right)X - \left(\frac{3d_i - 4d_c + d_j}{L}\right)$$

$$S(0) = -\left(\frac{3d_i - 4d_c + d_j}{L}\right)$$

$$S(L) = \frac{4d_i - 8d_c + 4d_j - 3d_i + 4d_c - d_j}{L} = \frac{d_i - 4d_c + 3d_j}{L}$$

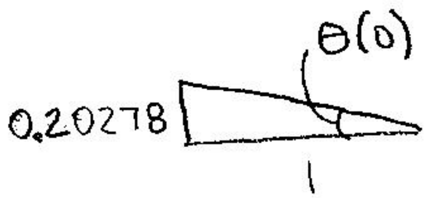
2. Calculate end slopes

$$S(0) = -\left(\frac{3\times10 - 4\times(-10)+3}{360}\right) = -0.20278 \text{ in/in}$$

$$S(L) = \frac{10 - 4\times(-10)+3\times3}{360} = 0.16389 \text{ in/in}$$

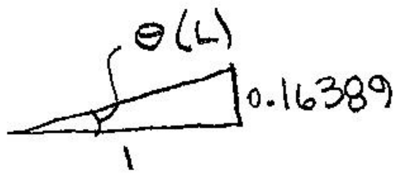
3. Calculate end angles

At L = 0":



$$\theta(0) = \text{Atan}(0.20278)$$
$$\theta(0) = 11.463^\circ$$

At L = 360":



$$\theta(L) = \text{Atan}(0.16389)$$
$$\theta(L) = 9.307^\circ$$

4. Determine friction loss coefficients at $L = 360''$

$$\text{Curvature Loss Coefficient} = \frac{0.15 \times (11.463^\circ + 9.307^\circ) \times \pi}{180}$$

$$= 0.05438$$

$$\text{Wobble Loss Coefficient} = 0.0001 \times 360'' = 0.036$$

5. Determine axial end forces before elastic shortening occurs

At $L = 0''$: $F_A = 220 \cos(11.463^\circ)$

$$F_A = 215.612^k$$

At $L = 360''$:

$$F_A = 220(1 - 0.05438 - 0.036)\cos(9.307^\circ)$$

$$F_A = 197.482^k$$

6. Estimate tension loss from axial shortening

P= average axial force

$$P = \frac{215.612 + 197.482}{2} = 206.5^k$$

$$\Delta_{conc} = \frac{PL}{E_{conc} A_{conc}}$$

$$\Delta_{conc} = \frac{206.5 \times 360}{3600 \times 18 \times 30} = 0.0382 \text{ in}$$

$$P_{tendon} = \frac{E_{tendon} A_{tendon} \Delta_{conc}}{L}$$

$$P_{tendon} = \frac{29000 \times 1.5 \times 0.0382}{360} = 4.62^k$$

Assume 4.6^k loss from axial shortening

$$\text{Stress loss} = \frac{4.6}{1.5} = 3.067 \text{ Ksi}$$

This loss is specified in Model A where the prestress is represented as loads.

7. Calculate end forces at $L = 0''$

$F_A = (220 - 4.6)\cos(11.463°) = 211.103^k \rightarrow$

$F_V = (220 - 4.6)\sin(11.463°) = 42.808^k \downarrow$

$M = 10F_A = 211.103 \times 10 = 2111.03$ k-in

8. Calculate end forces at $L = 360''$

$F_A = [\{220(1 - 0.05438 - 0.036)\} - 4.6]\cos(9.307°)$

$F_A = 192.943^k \leftarrow$

$F_V = [\{220(1 - 0.05438 - 0.036)\} - 4.6]\sin(9.307°)$

$F_V = 31.620^k \downarrow$

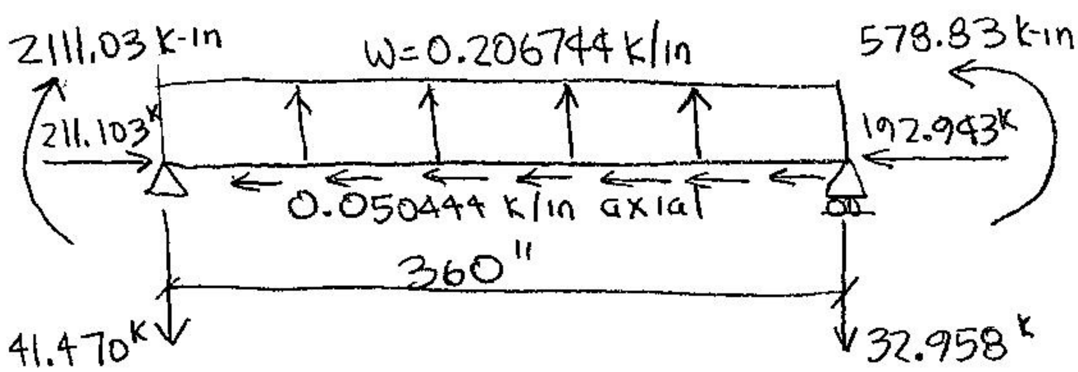
$M = 3F_A = 3 \times 192.943 = 578.83$ k-in

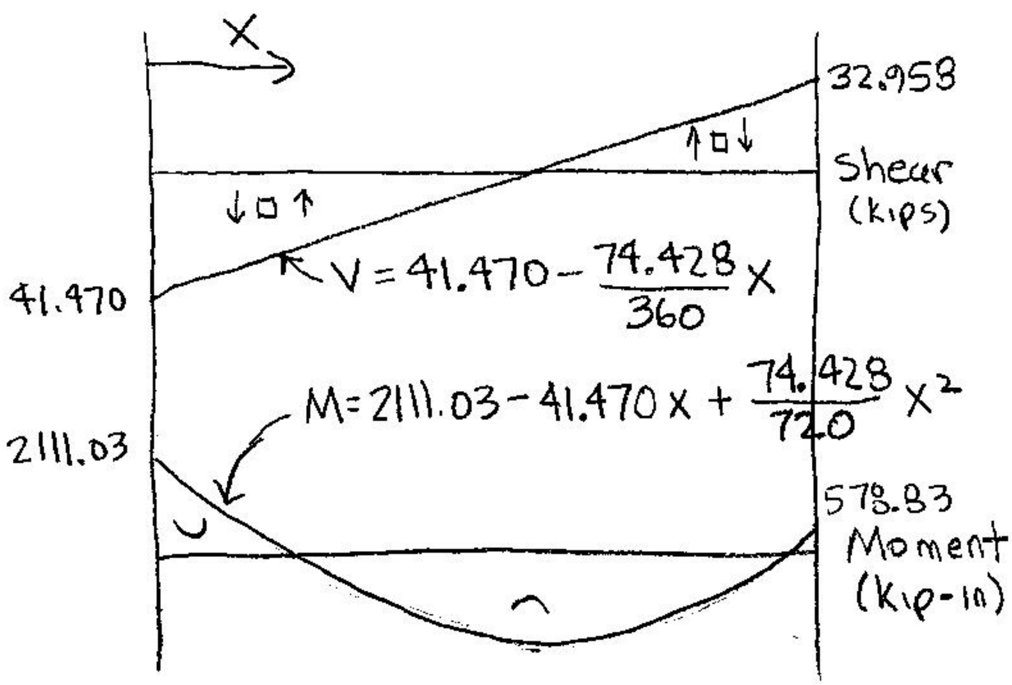
9. Determine magnitude of upward distributed load due to prestress. For simplicity in this hand calculation assume the distributed load is uniform.

$$\omega = \frac{(F_V @ L=0) + (F_V @ L=360)}{L} = \frac{42.808 + 31.620}{360}$$

$\omega = 0.206744$ K/in $\uparrow$

10. Plot equivalent loading on beam. Note that assumed uniform upward load from prestress yields calculated vertical reactions that are slightly different from previously calculated end forces





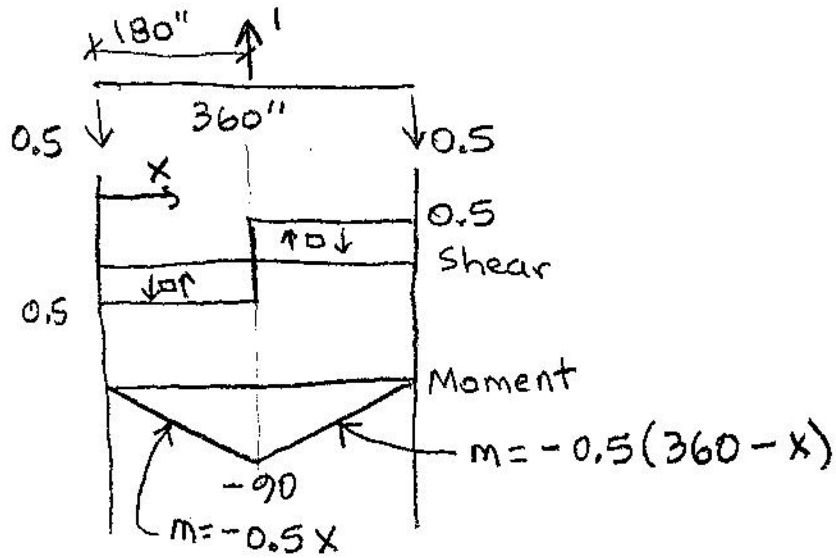
11. Calculate midspan deflection caused by prestress alone considering flexural and shear deformations.

Use $\Delta = \int \frac{mM}{EI} dx + \int \frac{vV}{GA_v} dx$

$$EI = \frac{3600 \times 18 \times 30^3}{12} = 145{,}800{,}000$$

$$GA_v = 1500 \times \frac{5}{6} \times 18 \times 30 = 675{,}000$$

Virtual System:



Flexural Deformation:

$$\Delta_f = \frac{1}{EI}\int_0^{180}\left(-1055.515x + 20.735x^2 - \frac{74.428}{1440}x^3\right)dx$$

$$+ \frac{1}{EI}\int_{180}^{360}\left(-379985.4 + 7464.6x - \frac{74.428}{4}x^2\right)dx$$

$$- \frac{1}{EI}\int_{180}^{360}\left(-1055.515x + 20.735x^2 - \frac{74.428}{1440}x^3\right)dx$$

$$\Delta_f = \frac{1}{EI}\left(\frac{-1055.515x^2}{2} + \frac{20.735x^3}{3} - \frac{74.428x^4}{5760}\right)\Bigg|_0^{180}$$

$$+ \frac{1}{EI}\left(-379985.4x + \frac{7464.6x}{2} - \frac{74.428x^3}{12}\right)\Bigg|_{180}^{360}$$

$$- \frac{1}{EI}\left(\frac{-1055.515x^2}{2} + \frac{20.735x^3}{3} - \frac{74.428x^4}{5760}\right)\Bigg|_{180}^{360}$$

$$\Delta_f = -0.117279 + 0.276467 - 0.093035$$
$$-0.938236 + 3.3176 - 1.984747$$
$$+0.469118 - 0.8294 + 0.248093$$
$$+0.469118 - 2.211733 + 1.48856$$
$$-0.117279 + 0.276467 - 0.093035$$

$\Delta_f = 0.160679$ in $\uparrow$, flexural deformation

Shear Deformation:

$$\Delta_v = \frac{1}{GA_v}\int_0^{180}\left(\frac{41.47}{2} - \frac{74.428}{720}x\right)dx$$
$$+ \frac{1}{GA_v}\int_{180}^{360}\left(-\frac{41.47}{2} + \frac{74.428}{720}x\right)dx$$

$$\Delta_v = \frac{1}{GA_v}\left(\frac{41.47x}{2} - \frac{74.428x^2}{1440}\right)\Bigg|_0^{180}$$
$$+ \frac{1}{GA_v}\left(-\frac{41.47x}{2} + \frac{74.428x^2}{1440}\right)\Bigg|_{180}^{360}$$

$$\Delta_v = 0.005529 - 0.002481$$
$$- 0.011059 + 0.009924$$
$$+ 0.005529 - 0.002481$$

$\Delta_v = 0.004961$ in ↑ , shear deformation

Total Deformation:

$$\Delta_{total} = \Delta_f + \Delta_v = 0.160679 + 0.004961$$

$$\underline{\underline{\Delta_{total} = 0.1564 \text{ in } \uparrow}}$$

12. Calculate center moment caused by prestress alone

$$M_{center} = 2111.03 - 41.47 \times 180 + \frac{74.428}{720} \times 180^2$$

$$M_{center} = -2004.3 \text{ K-in}$$

13. Calculate top and bottom S11 stress at midspan caused by prestress alone.

$$P_{center} = \frac{-211.103-192.943}{2} = -202.023^{K}$$
compression

$$\frac{P}{A} = \frac{P_{center}}{A_{conc}} = \frac{-202.023}{18 \times 30} = -0.3741 \text{ ksi}$$

$$I = \frac{bd^3}{12} = \frac{18 \times 30^3}{12} = 40{,}500 \text{ in}^4$$

$$\frac{Mc}{I} = \frac{-2004.3 \times 15}{40500} = -0.7422 \text{ ksi}$$

Top Stress:

$$\frac{P}{A} - \frac{Mc}{I} = -0.3741 + 0.7422 = 0.3681 \text{ ksi}$$
tension

Bottom Stress:

$$\frac{P}{A} + \frac{Mc}{I} = -0.3741 - 0.7422 = -1.1163 \text{ ksi}$$
compression