算例 1-026

框架 – 弯矩和剪力塑性铰

例题注释

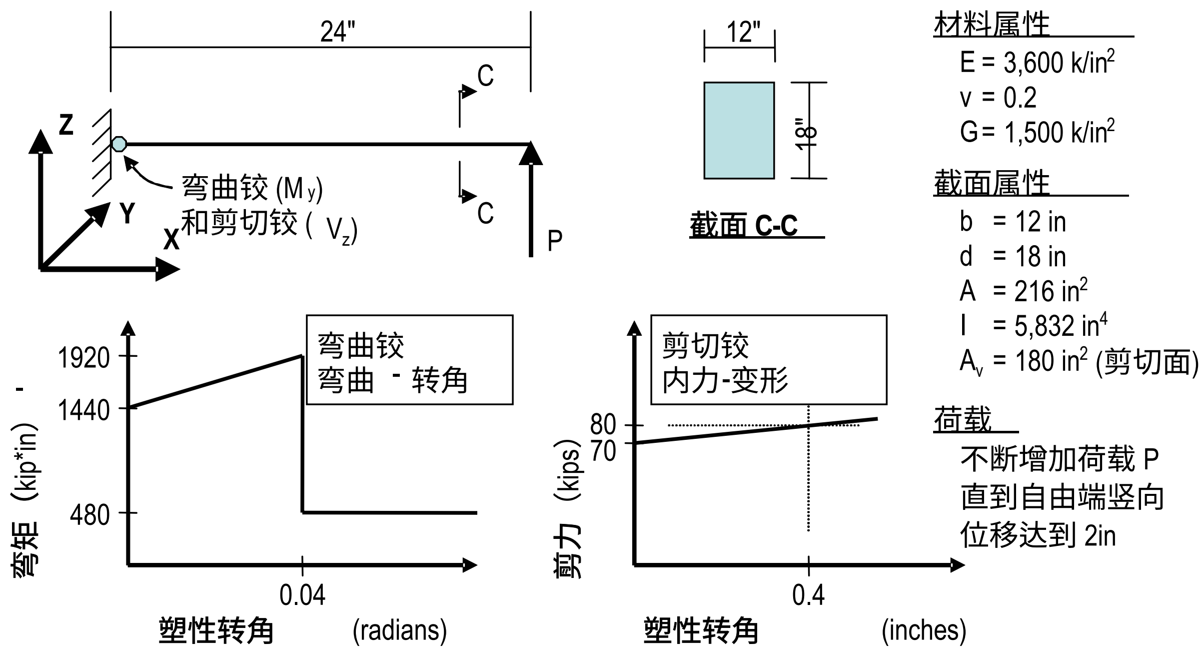
本例使用了一个水平悬臂梁来检验 SAP2000 静力非线性设计中的弯矩和剪力塑性铰。悬臂梁的固定端弯矩(M_y)铰和剪力(V_z)铰。一个竖向荷载 P 施加在悬臂端，其荷载值一直增加直到悬臂端竖向变形 U_z 等于 2”为止。

为了分析，多个模态被保存，设置最小模态数为 6，设置最大模态数为 10。顶部变形为 U_z，顶部扭转为 R_y，对于多个被保存的模态（参见下一页）结果与手算结果进行了比较。

重要注释:本例中包含了弯曲和剪力变形。

重要注释:SAP2000 中，框架铰只在非线性静力分析和非线性直接积分时程分析工况中才有效。铰在其它分析工况中将被忽略。

几何、属性和荷载参数

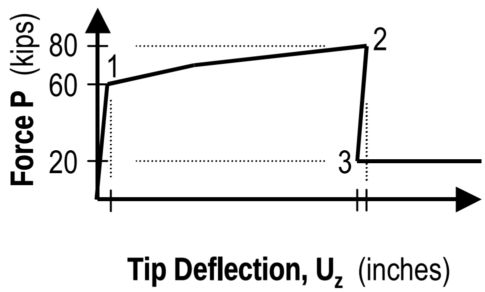


校验的 SAP2000 的技术特色

- 带有弯矩和剪力铰的非线性静力分析。

结果对比

输出的保存模态中荷载 P 和自由端位移和转角报告与右边所示悬臂梁力－顶点位移(P-U_z)图中节点标签 1、2 和 3 一致。手算结果是根据 Cook and Young （1985）244 页所描述的单位荷载方法和基本变形公式及叠加原理来得到的。



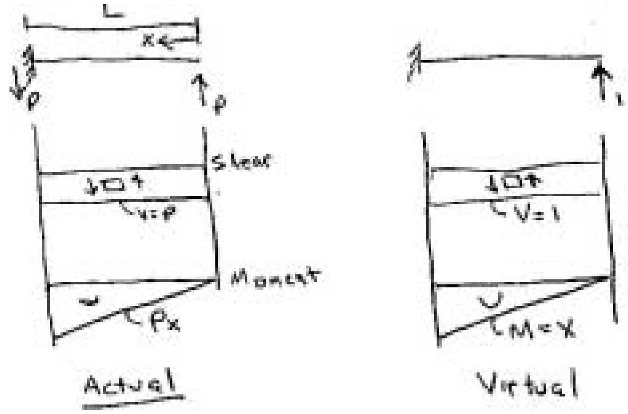
节点号	输出参数	SAP2000	手算解	差异百分比
1	力 P (自由端) kips	60	60	0%
	U_z (自由端) in	0.0185	0.0185	0%
	R_y (自由端) rad	-0.0008	-0.0008	0%
2	力 P (自由端) kips	80	80	0%
	U_z (自由端) in	1.3847	1.3847	0%
	R_y (自由端) rad	-0.0411	-0.0411	0%
3	力 P (自由端) kips	20	20	0%
	U_z (自由端) in	1.3662	1.3662	0%
	R_y (自由端) rad	-0.0403	-0.0403	0%

计算模型文件：Example 1-026

结论

SAP2000 结果与手算结果完全一致。

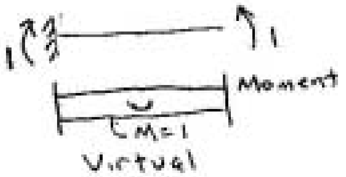
手算过程



$$\Delta = \int_0^L \frac{Mm}{EI}dx + \int_0^L \frac{Vv}{GA_v}dx = \int_0^L \frac{Px^2}{EI}dx + \int_0^L \frac{P}{GA_v}dx$$

$$\Delta = \frac{Px^3}{3EI}\Big|_0^L + \frac{Px}{GA_v}\Big|_0^L$$

$$\Delta = \frac{PL^3}{3EI} + \frac{PL}{GA_v} =$$ Equation For Elastic Uz deflection at cantilever tip



$$\Theta = \int_0^L \frac{Mm}{EI}dx = \int_0^L \frac{Px}{EI}dx = \frac{Px^2}{2EI}\Big|_0^L = \frac{PL^2}{2EI} =$$ Equation for elastic Ry rotation at cantilever tip

At Point 1

Elastic deformation only

$$U_z = \frac{PL^3}{3EI} + \frac{PL}{GA_v} = \frac{60 \times 24^3}{3 \times 3600 \times 5832} + \frac{60 \times 24}{1500 \times 180}$$

$$U_z = 0.01317 + 0.00533$$

$$U_z = 0.01850 \text{ in} \uparrow$$

$$R_y = \frac{PL^2}{2EI} = \frac{60 \times 24^2}{2 \times 3600 \times 5832} = 0.00082 \text{ rad} \circlearrowright$$

At Point 2

Moment hinge yielded at 60^k and Shear hinge yielded at 70^k

$$\text{Total Deformation} = \text{Elastic} + \text{Moment Hinge Plastic} + \text{Shear Hinge Plastic}$$

$$U_z \text{ elastic} = \frac{PL^3}{3EI} + \frac{PL}{GA_w} = \frac{80 \times 24^3}{3 \times 3600 \times 5832} + \frac{80 \times 24}{1500 \times 180}$$

$$U_z \text{ elastic} = 0.02467 \text{ in} \uparrow$$

$$U_{z \text{ plastic moment hinge}} = 24 \times 0.04 = 0.96 \text{ in} \uparrow$$

SAP2000 assumes small displacements here, thus $\sin\theta = \theta$

$$U_{z \text{ plastic shear hinge}} = 0.4 \text{ in} \uparrow$$

$$U_{z \text{ total}} = 0.02467 + 0.96 + 0.4 = 1.38467 \text{ in} \uparrow$$

$$R_z\ \text{elastic} = \frac{PL^2}{2EI} = \frac{80 \times 24^2}{2 \times 3600 \times 5832} = 0.00110$$

$$R_z\ \text{plastic moment hinge} = 0.04$$

$$R_z\ \text{plastic shear hinge} = 0$$

$$R_z\ \text{total} = 0.00110 + 0.04 + 0 = \underline{\underline{0.04110\ \text{rad}}} \circlearrowleft$$

At Point 3

Plastic deformation remains and elastic deformation is reduced to that for P = 20 kips

$$U_z\ \text{elastic} = \frac{PL^3}{3EI} + \frac{PL}{GA_v} = \frac{20 \times 24^3}{3 \times 3600 \times 5832} + \frac{20 \times 24}{1500 \times 180}$$

$$U_z\ \text{elastic} = 0.00617\ \text{in} \uparrow$$

U_z plastic same as point 2

$$U_z\ \text{total} = 0.00617 + 0.96 + 0.4 = \underline{\underline{1.36617\ \text{in}}} \uparrow$$

$$R_z\ \text{elastic} = \frac{PL^2}{2EI} = \frac{20 \times 24^2}{2 \times 3600 \times 5832} = 0.00027 \circlearrowleft$$

R_z plastic same as point 2

$$R_z\ \text{total} = 0.00027 + 0.04 + 0 = \underline{\underline{0.04027\ \text{rad}}} \circlearrowleft$$