算例 1-005

框架 – 位移荷载

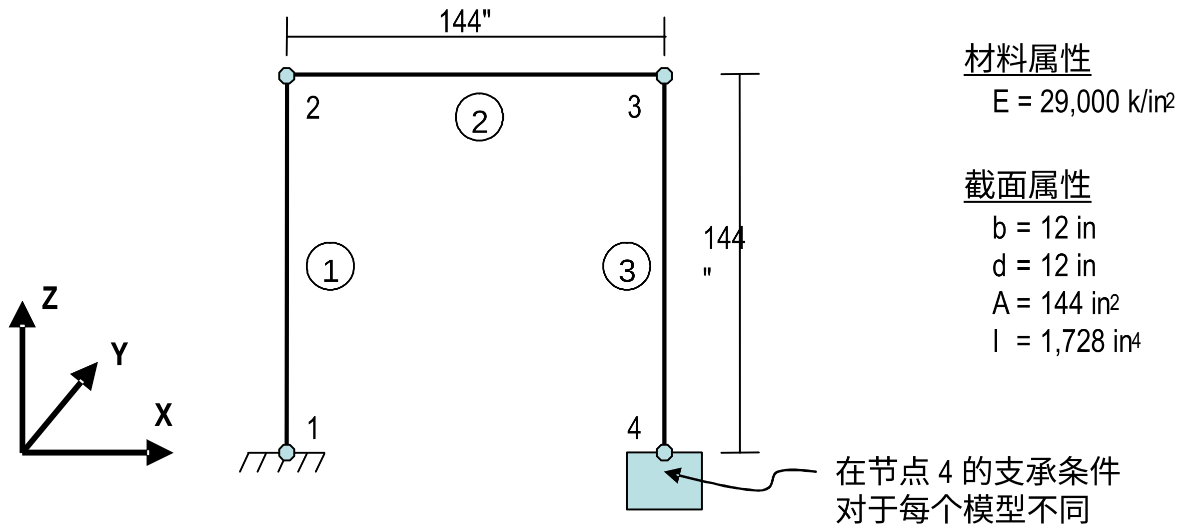
算例描述

本例采用一个门式框架验证了 SAP2000 中的一般支座、铰接支座（只是设置）和弹簧支座的设置和旋转的功能。注意对于弹簧支座，其接地端被移动或旋转了。

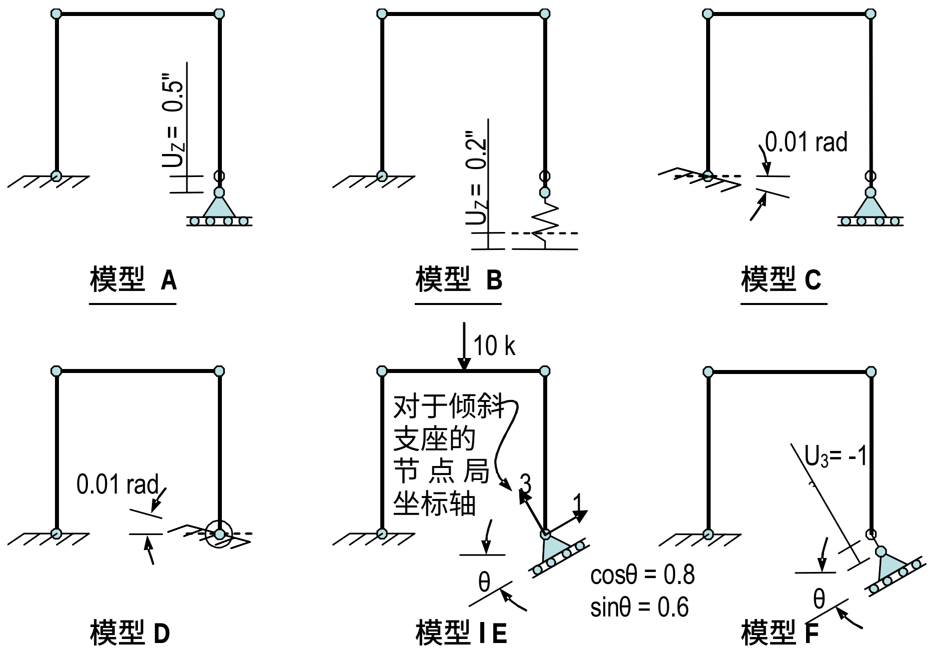
生成了六个不同的模型。除了如下图所示的节点 4 处的支撑情况不同外，这些模型是相同的。将不同支撑条件下的结果与手算结果进行了对本。

重要提示：分析中只考虑了弯曲变形。在 SAP2000 中，通过将面积属性修正参数设为 1000 并将剪切属性修正系数设置为 0 实现这一点。

几何特性和属性



节点 4 的支撑条件和荷载



模型	节点 4 的支撑条件	荷载
A	滑动支座	节点 4 的 Z 向位移-0.5"
B	Z 向平动弹簧，k = 10 kip/in	节点 4 的 Z 向位移-0.2"
C	滑动支座	节点 1 的旋转 0.01 弧度
D	绕 Y 轴的旋转弹簧， k = 80,000 kip-in/rad	节点 4 的旋转 0.01 弧度
E	滑动铰支座	框架单元 2 中点 Z 向外力 10 kip
F	滑动铰支座	节点 4 局部 3 轴方向位移-1"

所测试的 **SAP2000** 技术要点：

- 框架结构中支座的设置
- 框架结构中支座的旋转
- 线性（平动）弹簧的设置
- 旋转弹簧支座的转动
- 滑动支座
- 滑动支座的设置

结果比较

采用 Cook and Young 1985 一书第 244 页的单位力法计算手算得出独立结果。

模型	输出参数	SAP2000	独立结果	差值百分比
A. 支座沉降	F_z (节点 1) kip	6.293	6.293	0%
	M_y (节点 1) kip-in	-906.250	-906.250	0%
B. 弹簧支座沉降	F_z (节点 1) kip	1.115	1.115	0%
	M_y (节点 1) kip-in	-160.492	-160.492	0%
C. 支座旋转	F_z (节点 1) kip	-18.125	-18.125	0%
	M_y (节点 1) kip-in	2,610.000	2,610.000	0%
D. 弹簧支座旋转	M_y (节点 1) kip	-473.469	-473.469	0%
	R_y (节点 4) rad.	0.00408	0.00408	0%
E. 滑动支座	F_z (节点 1) kip	5.811	5.811	0%
	F_3 (节点 4) kip	5.236	5.236	0%

F. 滑动支座沉降	F_z (节点 1) kip	27.215	27.215	0%
	M_y (节点 1) kip-in	-3,918.919	-3,918.919	0%

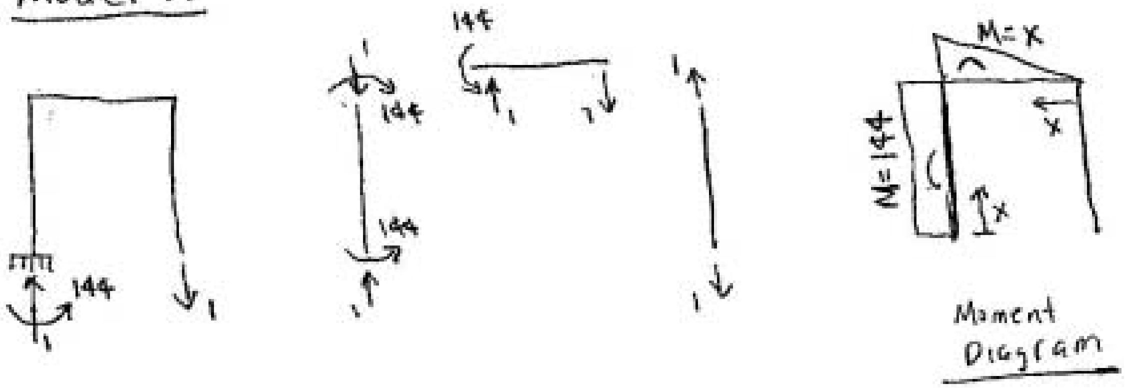
计算模型文件: 算例 1-005a, 算例 1-005b, 算例 1-005c, 算例 1-005d, 算例 1-005e, 算例 1-005f

结论

SAP2000 的结果和独立计算的结果精确地吻合。

手算：

Model A



$$\Delta_4 EI = \int_0^{144} 144^2 dx + \int_0^{144} x^2 dx$$

$$= 144^2 x \Big|_0^{144} + \frac{x^3}{3}\Big|_0^{144}$$

$$= 2985984 + 995328$$

$$\Delta_4 EI = 3981312$$

$$\Delta_4 = \frac{3981312}{EI} = \frac{3981312}{29000 \times 1728} = 0.079448 \downarrow$$

$$\text{Factor} = \frac{0.5}{0.079448} = 6.293403$$

$$F_z \text{ at jt } 1 = +1 \times 6.293403 = \underline{\underline{6.293^k}}$$

$$M_z \text{ at jt } 1 = -144 \times 6.293403 = \underline{\underline{-906.250}}$$

Model B

See Model A for analysis under unit load with the spring removed. For this case: $\Delta_4 = 0.079448 \downarrow$

Δ_g = ground displ
Δ_4 = joint 4 displ
Δ_s = spring stretch, K = 10 K/in

$$\Delta_s = \frac{F_{z@4}}{K} = \frac{\Delta_4}{0.079448K} = \frac{\Delta_4}{0.79448}$$

$$\Delta_4 + \Delta_s = \Delta_g = 0.2$$

$$\Delta_4 + \frac{\Delta_4}{0.79448} = 0.2$$

$$\Delta_4 = 0.2\left(\frac{0.79448}{1.79448}\right)$$

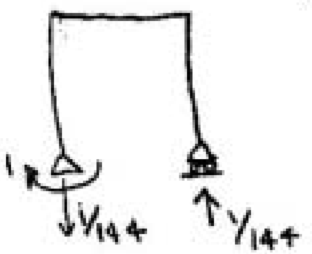
$$\Delta_4 = 0.088547$$

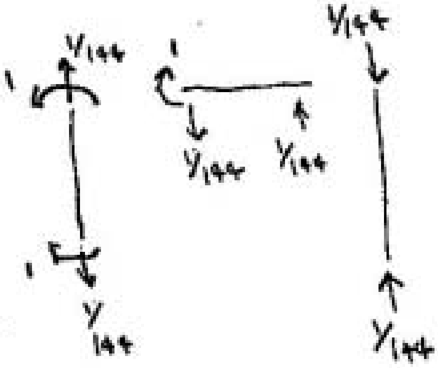
$$\text{Factor} = \frac{0.088547}{0.079448} = 1.114529$$

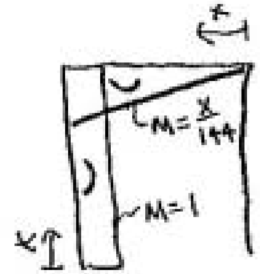
$$F_z \text{ at jt } 1 = +1 \times 1.114529 = \underline{\underline{1.115}}$$

$$M_z \text{ at jt } 1 = -144 \times 1.114529 = \underline{\underline{-160.492}}$$

Model C







Moment Diagram

$$\theta_1 EI = \int_0^{144} 1^2\,dx + \int_0^{144} \frac{x^2}{144^2}\,dx$$

$$= x\Big|_0^{144} + \frac{x^3}{3\times 144^2}\Big|_0^{144}$$

$$= 144 + \frac{144}{3}$$

$$= 192$$

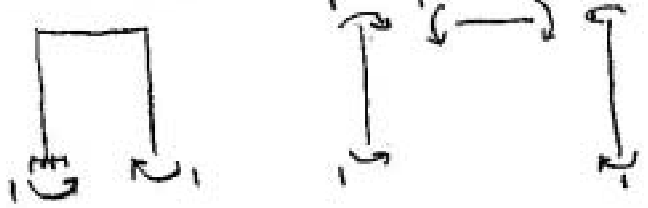
$$\theta_1 = \frac{192}{EI} = \frac{192}{29000\times 1728}$$

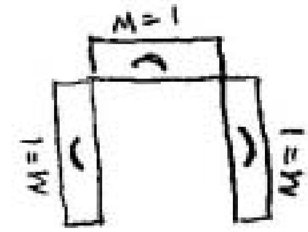
$$\text{Factor} = \frac{0.01}{\theta_1} = \frac{0.01\times 29000\times 1728}{192} = 2610$$

$$F_z \text{ @ jt } 1 = -\frac{1}{144}\times 2610 = -\underline{\underline{18.125}}$$

$$M_y \text{ @ jt } 1 = +1\times 2610 = \underline{\underline{2610}}$$

Model D





Moment Diagram

$$\theta_4 EI = 3\int_0^{144} 1^2 dx = 3x\Big|_0^{144} = 432$$

$$\theta_4 = \frac{432}{EI} = \frac{432}{29000 \times 1728}$$

ϕ_g = ground rotation

θ_4 = joint 4 rotation

θ_s = spring rotation, K = 80,000 k-in/rad

$$\theta_s = \frac{M_y @ 4}{K} = \frac{\theta_4 \times 29000 \times 1728}{432 \times 80000}$$

$$\theta_4 + \theta_s = \theta_g = 0.01$$

$$432 \times 80000 \times \theta_4 + 29000 \times 1728 \times \theta_4 = 0.01(432 \times 80000)$$

$$84672000\,\theta_4 = 345600$$

$$\theta_4 = \frac{345600}{84672000}$$

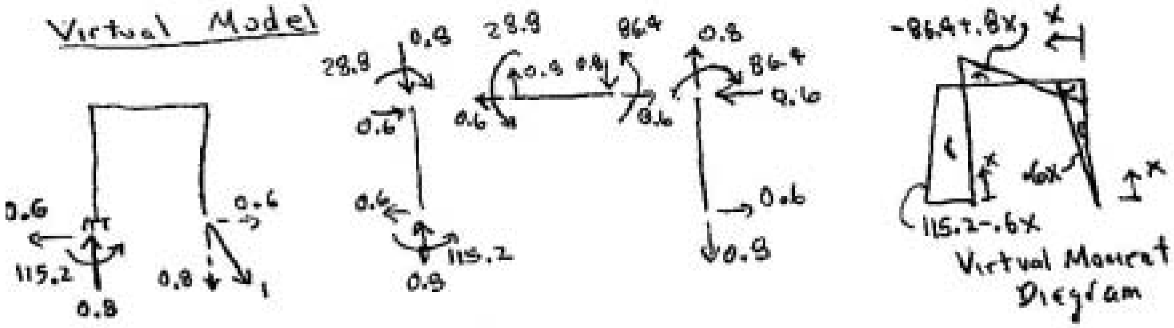
$$\text{Factor} = \frac{345600}{84672000} \times \frac{29000 \times 1728}{432} = 473.469398$$

$$M_y @ Jt\,1 = -1 \times 473.469398 = \underline{\underline{-473.469}}$$

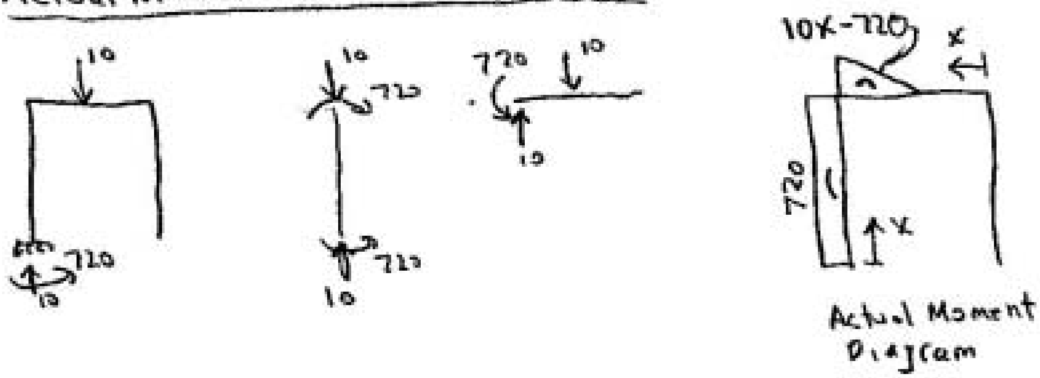
$$\theta_y \text{ at } Jt\,4 = \frac{432 \times 473.469398}{29000 \times 1728} = \underline{\underline{0.00408}}$$

Model E

Virtual Model



Actual Model With Roller Removed



Displacement of actual model with roller removed

$$\Delta_4 EI = \int_0^{144} (82944 - 432x)\,dx + \int_{72}^{144} (8x^2 - 1440x + 62208)\,dx$$

$$= 82944x\Big|_0^{144} - \frac{432}{2}x^2\Big|_0^{144} + \frac{8x^3}{3}\Big|_{72}^{144} - \frac{1440x^2}{2}\Big|_{72}^{144} + 62208x\Big|_{72}^{144}$$

$$= 11943936 - 4478976 + 7962624 - 995328 - 14929920 + 3732480 + 8957952 - 4478976$$

$$\Delta_4 EI = 7713792 \downarrow$$

Model E, cont

Displacement of model with unit load at roller

$$\Delta_4 EI = \int_0^{144} (0.36x^2 - 138.24x + 13271.04 + 0.64x^2 - 138.24x + 7464.96 + 0.36x^2)\,dx$$

$$\Delta_4 EI = \int_0^{144} (1.36x^2 - 276.48x + 20736)\,dx$$

$$= \frac{1.36x^3}{3}\Big|_0^{144} - \frac{276.48x^2}{2}\Big|_0^{144} + 20736x\Big|_0^{144}$$

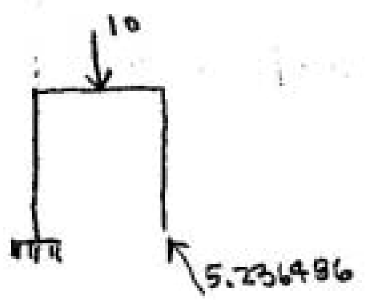
$$= 1353646.08 - 2866544.64 + 2985984$$

$$\Delta_4 EI = 1473085.44$$

Support Reactions

$$\text{Factor} = \frac{7713792}{1473085.44} = 5.236486$$

$$F_3 \text{ @ Jt } 4 = 1 \times 5.236486 = \underline{\underline{5.236}}$$



$$F_z \text{ @ Jt } 1 = 10 - 0.9(5.236486) = \underline{\underline{5.811}}$$

Model F

See Virtual model in Model E calculation

$\Delta_4 EI = 1473085.44$

$$\Delta_4 = \frac{1473085.44}{29000 \times 1728} = 0.029396 \downarrow$$

$$\text{Factor} = \frac{1}{0.029396} = 34.018393$$

F_z @ j+1 = +0.8 × 34.018393 = $\underline{\underline{27.215}}$

M_y @ j+1 = −115.2 × 34.018393 = $\underline{\underline{-3918.919}}$