**ELECTRICAL ENGINEERING**

TUNING A PI CONTROLLER FOR FIRST ORDER SYSTEMS

1 TRANSFER FUNCTION

The model of a first order system is the simplest one you will find. It is used for numerous circuits when the dead time is negligible in relation to the time constant. It is important to remember that such a system is always stable.
However we can link a controller to the open loop transfer function in order to adjust the precision and the response time in a closed loop.

The transfer function of the process is of the type :

$$H(s) = \frac{K}{1 + \tau s}$$

(K : static gain)
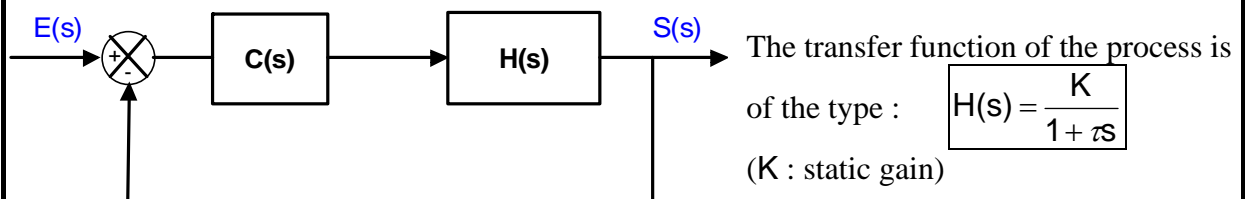
*Reminder***: we measure the time constant τ, at 63 % of the full-scale output value.**

A PI type control structure will allow us to obtain almost perfect static precision and to adjust the response time.

We therefore posit

$$C(s) = A\left(1 + \frac{1}{T_i s}\right) \quad (1.1)$$

A : proportional action coefficient
T_i : integration constant

The transfer function $F(s)$, for closed loop is :

$$F(s) = \frac{S(s)}{E(s)} = \frac{H(s).C(s)}{1 + H(s).C(s)} \quad (1.2)$$

That we can write in the form of a second order :

$$F(s) = \frac{1 + T_i s}{1 + \frac{(KA + 1)}{KA} T_i s + \frac{T_i \tau}{KA} s^2} \quad (1.3)$$

that can be identified with a canonical form :

$$F(s) = \frac{1 + as}{1 + \frac{2\lambda}{\omega_0} s + \frac{1}{\omega_0^2} s^2} \quad (1.4)$$

We obtain through identification :

$$\lambda = \frac{1+KA}{2\sqrt{KA}} \sqrt{\frac{T_i}{\tau}} \quad (2.1) \qquad \text{and} \qquad \omega_0 = \sqrt{\frac{KA}{T_i \tau}} \quad (2.2)$$

The methods of tuning all use the product KA, so we define the circuit gain of the loop K_B by $K_B = KA$

2 TUNING

First method

- **Set the value of $\lambda = 1$ (standard tuning method)**
  The equality (2.1) allows us to link the values of T_i and K_B:

$$\boxed{T_i = \frac{4K_B \tau}{(1+K_B)^2}} \ (2.3)$$

  Moreover, we know that the response time can be estimated by $T_r \approx \dfrac{3}{\lambda \omega_0}$,

- **Set the response time**:
  By using (2.1) and (2.2), we obtain:

$$\boxed{T_r \approx \frac{6\tau}{1+K_B}} \ (2.4)$$

  The result is independent of T_i.

- **We can then deduce the value of T_i :**
  The selected response time allows us to determine the circuit gain of the loop K_B (therefore of A), we can then deduce the value of T_i (from the equality 2.3)

Second method

- **Set the value of T_i :**
  For tight tuning ($\lambda < 1$), we will use the value $T_i = 0.5\tau$
  For smoother tuning ($\lambda = 1$), we will use $T_i = 0.75\tau$
  For an underdamped response ($\lambda > 1$), we will use $T_i = \tau$

- **Set the response time:**
  The equality (2.4) allows us to calculate the gain of the loop K_B and to deduce the value of A.