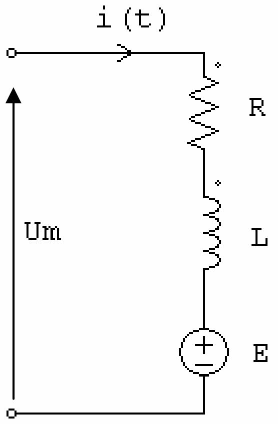
POWERSYS
ELECTRICAL ENGINEERING

MODELLING OF A DC MOTOR

1. Equations of the motor:

The modeling of the unit motor + load is made possible by using the basic equations of the DC machine and the fundamental principle of dynamics.
We propose to model a **permanent magnet machine** (with constant flux, no magnetic saturation).

You can see on the right the equivalent circuit of the armature of the machine.



We can deduce the following electrical equations in any particular state:

$$U_m(t) = Ri(t) + L\frac{di(t)}{dt} + E(t) \qquad (1.1)$$

(k, emf constant, considered as equal to the torque constant).

Here is a reminder of the electromagnetic equations linked to the motor :

$$E(t) = k\Omega(t) \;\text{ et }\; C_m(t) = ki(t)$$

The equality 1.1 can therefore be written:

$$\boxed{U_m(t) = Ri(t) + L\frac{di(t)}{dt} + k\Omega(t)} \qquad (1.2)$$

Mechanical equations:

C_m: the electromagnetic torque produced by the motor.
J: is the moment of inertia.
C_{ch}: torque load. It consists in the load torque C_r and the friction C_f.
With $C_f = f\Omega$ (we will ignore dry friction).
f: coefficient of viscous friction.
The fundamental principle of dynamics allows us to write:

$$J\frac{d\Omega(t)}{dt} = C_m(t) - C_{ch}(t) \qquad (1.3)$$

By stating that $C_{ch} = C_r + C_f$, the equation (1.3) can be written this way:

$$\boxed{C_m(t) = J\frac{d\Omega(t)}{dt} + f\Omega + C_r(t)} \qquad (1.4)$$

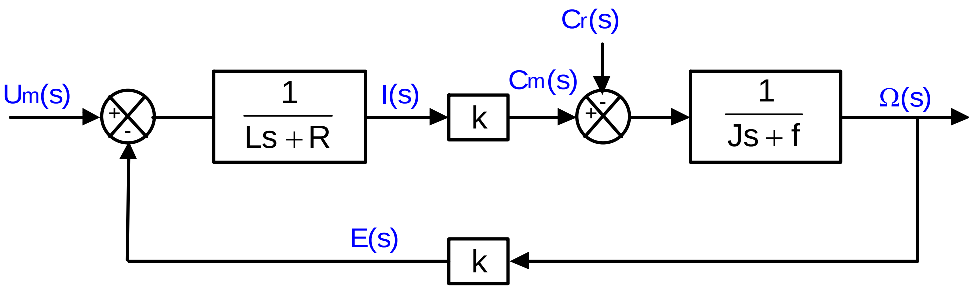
2. Block diagram :

We will now look at variations in the armature current and the speed (the quantities studied are considered to be nil at the initial instant**).**
In LAPLACE variables the equations (1.2) and (1.4) become:

$$U_m(s) = (R + Ls)I(s) + k\Omega(s) \quad \text{et} \quad C_m(s) = (f + Js)\Omega(s) + C_r(s)$$

We therefore deduce: $\boxed{I(s) = \dfrac{U_m(s) - k\Omega(s)}{Ls + R}}$ and $\boxed{\Omega(s) = \dfrac{kI(s) - C_r(s)}{Js + f}}$

We obtain the block diagram:



3. Transfer functions:

3.1. **Control of the speed by the armature**:
After calculating we find:

$$\Omega(s) = k\frac{U_m(s)}{(Js + f)(Ls + R) + k^2} - C_r(s)\frac{(Ls + R)}{(Js + f)(Ls + R) + k^2}$$

In the case of speed control, we consider the load torque as a disturbance and the transfer function in voltage is written:

$$\boxed{T_\Omega(s) = \frac{\Omega(s)}{U_m(s)} = \frac{k}{(Js + f)(Ls + R) + k^2}} \quad (2.1)$$

We find a second order transfer function. We posit usually:
$\tau_e = \dfrac{L}{R}$ and $\tau_m = \dfrac{J}{f}$ and we define the electromechanical time constant: $\tau_{em} = \dfrac{Rf}{k^2 + Rf}\tau_m$

and the coefficient K_m so that $K_m = \dfrac{k}{k^2 + Rf}$

We can approach the relation(2.1) in the domain of common frequencies by :

$$\boxed{T_\Omega(s) = \frac{1}{K_m}.\frac{1}{(1 + \tau_e s)(1 + \tau_{em} s)}} \quad (2.2)$$

When τ_e is very small in relation to τ_{em}, or several frequencies being envisaged are very weak, we can reduce (2.2) to :

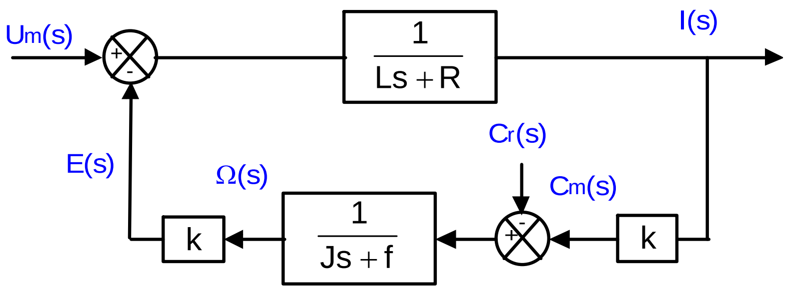
$$\boxed{T_\Omega(s) = \frac{1}{K_m} \cdot \frac{1}{(1+\tau_{em}s)}} \quad (3.1)$$

In this case the machine is seen as a simple first order (however it must not be subject to stress to quickly…).

Note: a variation in the load causes a modification in the model of the system. Moreover, if the load is large the simplification hypotheses used to obtain (2.2) can turn out to be unsound…

3.2. **Current control by the armature**:

The block diagram on the previous page can be represented in the following manner:



This block diagram will be used to study current control (or torque control). It will also be useful to set the parameters of the current circuit in the case of speed control with a secondary loop.

If we consider that the torque load is a disturbance, the transfer function in current is written:

$$\boxed{T_I(s) = \frac{I(s)}{U_m(s)} = \frac{Js+f}{(Js+f)(Ls+R)+k^2}} \quad (3.2)$$

We can show that this relation can be approached by:

$$\boxed{T_I(s) = A.\frac{(1+\tau_m s)}{(1+\tau_e s)(1+\tau_{em} s)}} \text{ avec } A = \frac{f}{Rf+k^2} \quad (3.3)$$

The remarks are the same as in the previous chapter. Generally, $\tau_m > \tau_{em} > \tau_e$ and we can show that in a closed loop the transfer function is equivalent to that of a first order.